

#### SIZE AND SHAPE PARAMETERS DEFINED IN THE PSA300

At the moment there is minimal standardization for defining particle size shape when using automated image analysis. Although particle size distribution calculations are defined in several standards (1, 2), few companies currently used the ASTM standards for particle shape calculations (3). This document defines the various particle size and shape parameters calculated in the PSA300 software.

#### Anisotropy

An estimate of the extent to which a bitplane can be said to be elongated:

$$\frac{\text{Mean Vertical Chord}}{\text{Mean Horizontal Chord}}$$

#### Average Area (of Objects)

The mean area covered by objects belonging to a selected bitplane:

$$\frac{\text{Total Area}}{\text{Count}}$$

#### Centroids

The centroid of a field is the point on which it would be possible to balance a bitplane on the tip of a needle. The centroid of a set of n pixels belonging to an object at position  $X_i$  is:

$$\bar{x} = \frac{\sum_{i=1}^n X_i}{n}$$

#### Circular Diameter

Estimate of an object's circularity:

$$\text{Mean chord} \times 1.27324$$

#### Density

Density of objects within a given area.

$$\frac{\text{Count}}{\text{Field Area}}$$

#### Intensity

Average gray value of pixels belonging to a selected bitplane:

$$\frac{\sum \text{pixel gray intensity values}}{\text{Total number of pixels in the processing frame}}$$

#### Mean Horizontal Chord

Approximates the width of objects:

$$\frac{\sum \text{horizontal detected test line lengths}}{\sum \text{horizontal intercepts}}$$

#### Spherical Diameter

Estimate of the size of an object as if it was a sphere:

$$\text{Mean chord} \times 1.27324 \times 1.2247$$

#### Aspect Ratio

Ratio of length over width.

$$\frac{\text{Length of longest feret}}{\text{Length of shortest feret}} = \frac{\text{Length}}{\text{Width}}$$

#### Moment

The Binary moment measures provide a geometrical description of the object.

$$f(x, y) \rightarrow \text{Bitplane Value at position } X \text{ or } Y (0 \text{ or } 1)$$

1<sup>st</sup> order moment in X & Y

$$m_{11} = \sum_x \sum_y (x - x_{Avg}) \times (y - y_{Avg}) \times f(x, y)$$

2<sup>nd</sup> order moment in X

$$m_{11} = \sum_x \sum_y (x - x_{Avg})^2 \times f(x, y)$$

2<sup>nd</sup> order moment in Y

$$m_{11} = \sum_x \sum_y (y - y_{Avg})^2 \times f(x, y)$$

### Center of Mass in X

The centroid of a field (or center of mass) is the point on which it would be possible to balance a field on the tip of a needle.

The centroid of a set of n pixels belonging to an object at position  $X_i$  is:

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

### Center of Mass in Y

The centroid of a field (or center of mass) is the point on which it would be possible to balance a field on the tip of a needle.

The centroid of a set of n pixels belonging to an object at position  $Y_i$  is:

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n}$$

### Circular Diameter

Diameter estimate (as if the feature was a two dimensional object).

$$2\sqrt{\frac{\text{area}}{\pi}}$$

### Compactness

Ratio of area over convex perimeter:

$$\frac{4\pi A}{\text{Convex perimeter}^2}$$

### Convex Perimeter

Line joining feret tangent points:

$$\sum \text{ferets } (2\text{tan}) \left( \frac{\pi}{2(\text{number of ferets})} \right)$$

### Fractal Dimension

Numerical characterization of irregular contours through fractal geometry.

$$P = P_E \delta^{1-D}$$

D is the Fractal Dimension,  $\delta$  is the unit length of the scale used for the measurement and P is the perimeter of the object ( $1 < D < 2$ ).

### Intensity

Average gray value of pixels belonging to an object (scaled from 0-100).

$$\frac{\sum \text{pixel gray values}}{\text{Total number of pixels}}$$

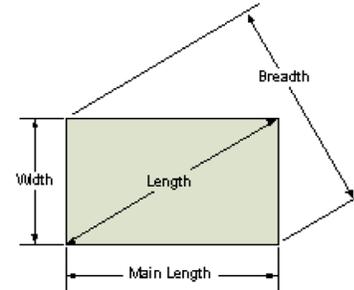
### Intensity Standard Deviation

Standard deviation of gray level intensity inside each object (scaled from 0-100).

$$Isd = \sqrt{\sum \text{object pixels } [( \text{pixel Int.} - \text{avg object Int.} )^2 \cdot \text{pixel Int.}]}$$

### Main Length

Length of the feret perpendicular to the shortest feret.



### Roughness

A shape measure that quantifies the jaggedness of an object's edges:

$$\frac{\text{Convex perimeter}}{\text{Perimeter}}$$

### Roundness

A shape measure that quantifies the "roundness" of an object's edges:

$$\frac{4 \times \text{Area}}{(\pi \times L \times L)}$$

### Spherical Diameter

Estimate equivalent to the diameter of a three dimensional object:

$$2 \times 1.2247 \times \sqrt{\frac{\text{area}}{\pi}}$$

### Sphericity

Estimate of the sphericity of an object:

$$\frac{4\pi A}{p^2}$$

### String Aspect Ratio

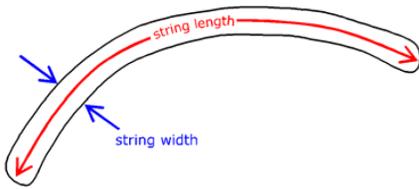
Shape factor of a thin curved object, expressing how many times it is longer than wider:

$$\text{String Aspect Ratio} = \frac{\text{String Length}}{\text{String Width}}$$

### String Length

Length of a thin curved object, measured along its medial axis. String length is approximated by:

$$\text{String Length} = \frac{\text{perimeter} + \sqrt{\text{perimeter}^2 - (16 \times \text{area})}}{4}$$



### String Width

Width of a thin curved object, measured across its medial axis. The object is assumed to have a constant width. String width is approximated by:

$$\text{String Width} = \frac{\text{perimeter} - \sqrt{\text{perimeter}^2 - (16 \times \text{area})}}{4}$$

### Volume, Cylindrical

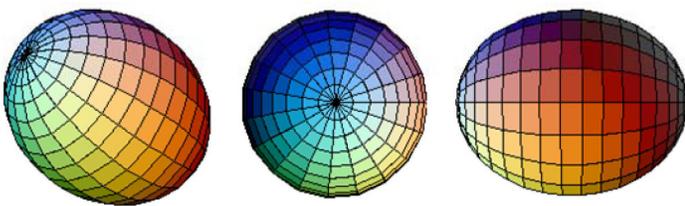
Estimated volume of a cylinder based on its side view in a 2D image (rectangle). Cylindricity is assumed by knowledge of the sample.

$$V_{\text{cyl}} = \frac{\pi}{4} \frac{\text{Area}^2}{\text{Main length}}$$

### Volume, Ellipsoidal

Estimated volume of a prolate spheroid based on its side view in a 2D image (ellipse). Cross-section of the solid is assumed to be a circle.

$$V_{\text{ell}} = \frac{\pi}{6} \text{Main length} \times \text{Width}^2$$



Left and center: 3D assumption; Right: 2D view.

### Volume, Spherical

Estimated volume of a sphere based on its view in a 2D image (circle). Sphericity is assumed by knowledge of the sample.

$$V_{\text{sph}} = \frac{\pi}{6} \text{Circular diameter}^3$$

### Volume, Tetragonal

Estimated volume of a rectangular prism based on its side view in a 2D image (rectangle). Cross-section of the solid is assumed to be a square.

$$V_{\text{tetr}} = \frac{\text{Area}^2}{\text{Main length}}$$

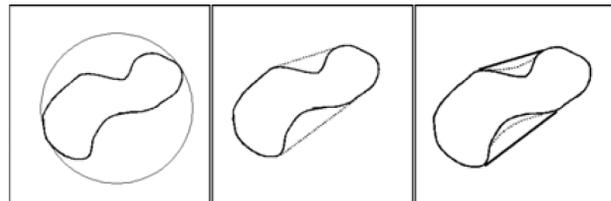
### Area Percent

Area Percent measurements are total area measurements not based on stereology. They express size as a percentage of a reference area and are cumulative for a run.

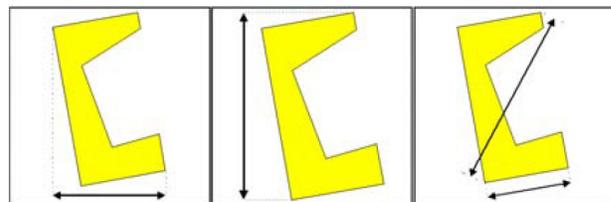
Area of a bitplane relative to the total process frame or reference bitplane:

$$\frac{\text{Area of bitplane} \times}{\text{Total area of process or reference bitplane}} \times 100$$

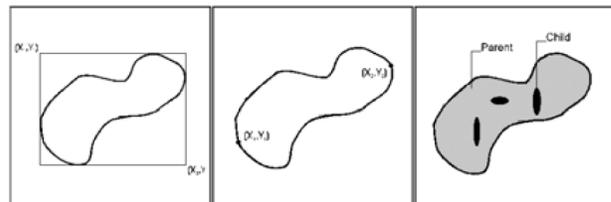
### Visual Reference



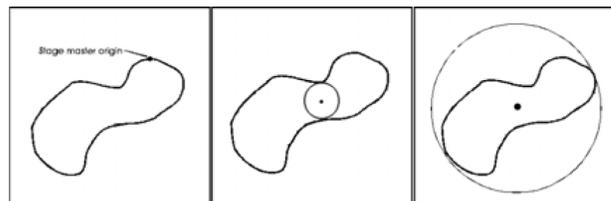
Roundness (left), Convex Perimeter (center), Roughness (right).



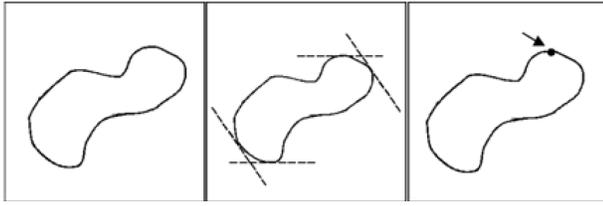
Feret 0° (left), Feret 90° (center), Aspect Ratio (right).



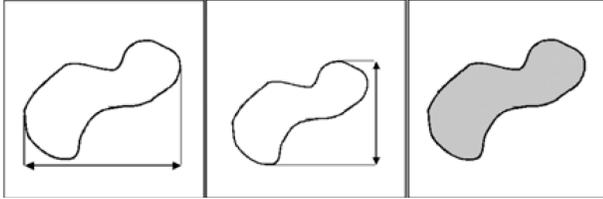
Bounding Rectangle, Extreme Coordinates, Parent/Child.



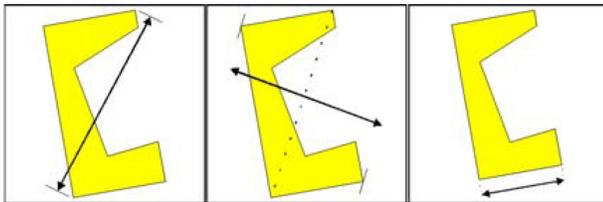
Distance (left), Inner Distance (center), Outer Distance (right).



**Perimeter (left), Ferets (center), Object Count Point (right).**



**X Projection (left), Y Projection (center), Area (right).**



**Length (left), Breadth (center), Width (right).**

## References

1. ASTM E2578-07 Standard Practice for Calculation of Mean Sizes/Parameters and Standard Deviations of Particle Size Distributions
2. ISO 9276-2 Representation of results of particle size analysis -- Part 2: Calculation of average particle sizes/ diameters and moments from particle size distributions
3. ASTM F 1877 Standard Practice for Characterization of Particles



**The HORIBA PSA300**