# PI Parameter Optimization for Pressure-Based MFC (D700) Using Gaussian Mixture Model

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A vast number of mass flow controllers (MFCs) are used in semiconductor industry. An efficient production method of MFC is required. The gain tuning of the proportional-integral (PI) control to realize a setting flow rate is essential for efficient mass production. The gains are tuned to meet the specifications required for evaluation indices of response time and overshoot amount in a step response waveform. In this paper, we propose a simple method for the PI gain tuning using the Gaussian Mixture Model (GMM) and the direct inverse analysis applicable to the pressure-based MFCs' production. The relationship between the gains and evaluation indices for a standard unit of the MFC is modeled as the GMM. The direct inverse analysis calculates the difference between the standard and a test unit. Under the assumption that the difference can be compensated by a simple shift, gains likely to meet the specifications for the test unit are searched. We applied the method to seven test units. The result showed that the gains of all the test units were tuned within only a few iterations.

Keywords
Semiconductor, Gaussian mixture mode, PI control, Mass flow controller, Manufacturing.

## 1. Introduction

A mass flow controller (MFC), which precisely controls fluid mass flow rate, is widely used in the semiconductor manufacturing[1]. Since a whole process of semiconductor manufacturing consists of many reaction steps, a vast number of MFCs are required for each plant. Therefore, establishing an efficient production method of MFC has become more important.

The MFC controls fluid's flow rate by adjusting a valve opening to realize a setting flow rate. The pressure-based MFC, which measures flow rate with pressure sensors, has advantages on its fast response and high accuracy.

The design of controller including an algorithm for the adjustment is essential for the MFCs' production. The proportional-integral (PI) or proportional-integral-derivative (PID) controls are widely employed for the algorithm, where the PI or PID gains determine an applied voltage to the valve so that the variation between the setting and measured flow rates can be converged. The gains are tuned to meet the specifications required for evaluation indices, which are a response time and an overshoot amount obtained from a step response waveform. In most MFCs' productions, the gains are tuned through trial and error for every unit, which is a bottleneck against establishing the efficient production method. The Gaussian mixture model (GMM) is a simple model to express a relationship among multiple variables with a superposition of Gaussian probability densities[2],[3]. The application can provide a simple model for the relationship between the gains and resulting evaluation indices. In addition, the direct inverse analysis based on the Bayes theorem enables us to estimate the most likely gains for obtained indices, conversely. However, as devices installed in the control target system have different characteristics among individual MFC units, the relationship modeled for a single unit cannot be simply applied for every unit.

In this paper, we propose a simple method for the PI gain tuning using the GMM and the direct inverse analysis applicable to the pressure-based MFCs' production. The relationship between the PI gains and evaluation indices are investigated for a standard unit. Using the GMM and the direct inverse analysis, the difference of every test unit from the standard unit is calculated. By compensating the difference, the gains are tuned to meet the specifications for every test unit. After formulating the method, we applied it to the tuning for a couple of pressure-based MFCs to confirm the applicability in mass production.

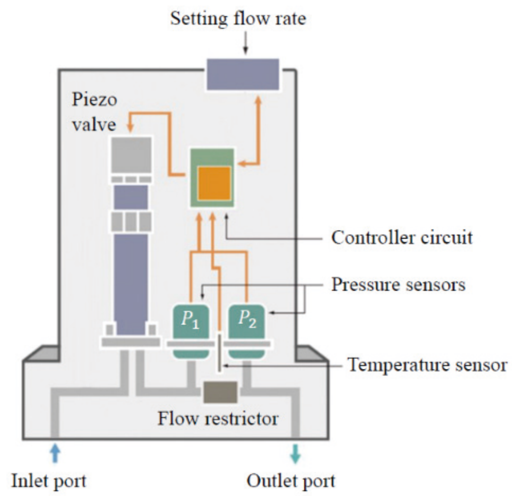


Figure 1 Internal structure of a pressure-based gas MFC.

## 2. CHARACTERISTICS OF PRESSURE-BASED MFCS

### 2.1 MFC structure

A schematic of a pressure-based gas MFC is shown in Figure 1. The MFC includes a piezo valve, two pressure sensors, a temperature sensor, a flow restrictor, and a controller circuit. The inlet and outlet ports for gas are respectively connected with upstream and downstream pipelines. Gas entered from the inlet port passes through the piezo valve and flow restrictor, and then goes out from the outlet port. The flow rate of output gas ($Q_{out}$) is calculated as follows:

$$Q_{out} = k(p_1^2 - p_2^2), \quad (1)$$

where $k$ is a flow restrictor constant having a temperature dependence, and $p_1$, $p_2$ are outputs of pressure sensors P1, P2, respectively.

The controller circuit adjusts the valve opening through applied voltage to realize a setting flow rate ($Q_{set}$). The block diagram of PI control installed in the controller is shown in Figure 2. Here, $K_P$, $K_I$ mean P and I gains,

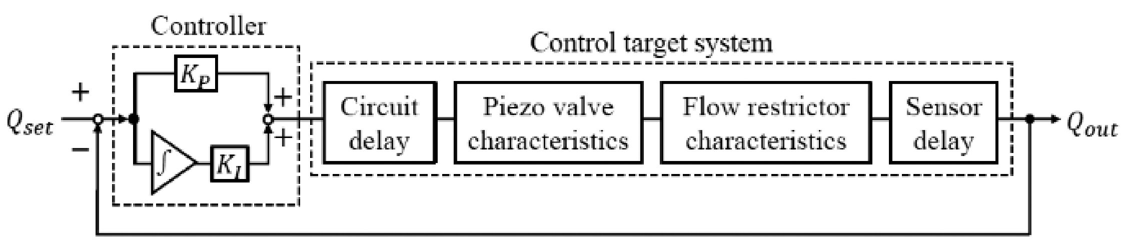


Figure 2 Block diagram of PI control system for the pressure-based MFC.

respectively. The gains are tuned in accordance with characteristics of the control target including circuit delay, characteristics of piezo valve and flow restrictor, and sensor delay, which are generally complicated. Furthermore, as the characteristics are different among individual units, every MFC unit requires individual gain tuning.

### 2.2 Evaluation indices for PI gain tuning

In this study, we consider the case that the PI gains are tuned based on the following two evaluation indices in a step response for 0→100%FS input ("%FS" means flow rate normalized by control full scale): (i) $T_r$(ms): response time defined as a period from the $Q_{set}$ input to the timing when $Q_{out}$ achieves 98% of $Q_{set}$ and (ii) $Q_{os}$ (%): overshoot amount defined as,

$$Q_{os} = \frac{Q_{peak} - Q_{set}}{Q_{set}} \times 100, \tag{2}$$

where $Q_{peak}$ is the flow rate having the maximum difference from $Q_{set}$ after once $Q_{out}$ exceeds $Q_{set}$. ($Q_{os}$ = 0 if $Q$ never exceeds $Q_{set}$) The total test duration for step response is fixed at 200 ms. The specifications require that $T_r$ and $Q_{os}$ should be within the range of 85 ± 5 ms and $0^{+0.50}_{-0.55}$ %, respectively. Here, we denote that the optimal indices ($T_{r0}$, $Q_{os0}$) = (85, 0) and the tolerances ($\Delta T_r$, $\Delta Q_{os}$) = (10, 1.05).

A global relationship between ($K_P$, $K_I$) and ($T_r$, $Q_{os}$) were investigated for a standard unit that are arbitrarily selected from among mass produced MFCs. We independently varied ($K_P$, $K_I$) in the range of [0.5,1.5] with intervals of 0.02, and thus 2,500 step response waveforms in total are acquired to get ($T_r$, $Q_{os}$). For the standard unit of this study, the optimal gains that gave the indices closest to ($T_{r0}$, $Q_{os0}$) were ($K_{P0}$, $K_{I0}$) = (0.94, 0.94). Figure 3 shows the step response waveforms at ($K_P$, $K_I$) = ($K_{P0}$, $K_{I0}$), (1.50, 0.50), and (0.50, 1.50). For ($K_P$, $K_I$) = ($K_{P0}$, $K_{I0}$), a preferable waveform with ($T_r$, $Q_{os}$) = (85, –0.046) were obtained. For ($K_P$, $K_I$) = (1.50, 0.50), the waveform had an oscillation with a large overshoot. For ($K_P$, $K_I$) = (0.50, 1.50), the waveform indicated a too slow response.

To simply evaluate the step response, we defined the deviation index $z$ as follows:

$$z = \frac{|T_r - T_{r0}|}{\Delta T_r} + \frac{|Q_{os}|}{\Delta Q_{os}}. \tag{3}$$

The necessary condition in which either index certainly meets the specifications is $z \le 1.02$, and the sufficient condition in which both indices certainly meet the specifications is $z \le 0.50$. For $0.50 < z \le 1.02$, the balance of each term in Eq. (3) determines whether the specifications are satisfied or not. The distribution of $z$ on a $K_P$ - $K_I$ plane for the standard unit is illustrated in Figure 4, together with regions of the necessary (white dashed line) and sufficient (white solid line) conditions and ($T_{r0}$, $Q_{os0}$) location (red spot). The PI gains likely to meet the specifications are configured in a narrow region around $K_P \approx K_I$. The distribution reflects the characteristics of the pressure-based MFC structure.

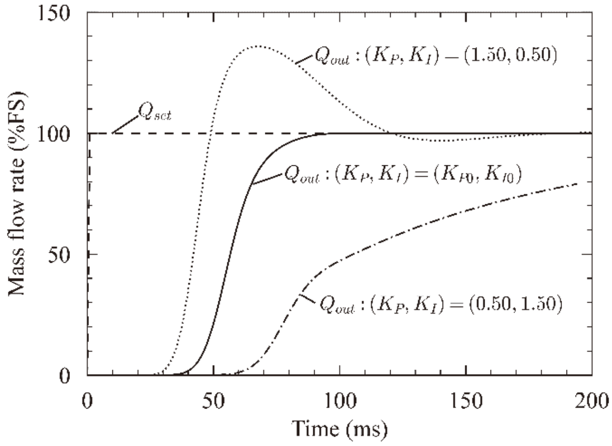


Figure 3 Step response waveforms of standard unit at ($K_P$, $K_I$) = ($K_{P0}$, $K_{I0}$), (1.50, 0.50) and (0.50, 1.50).

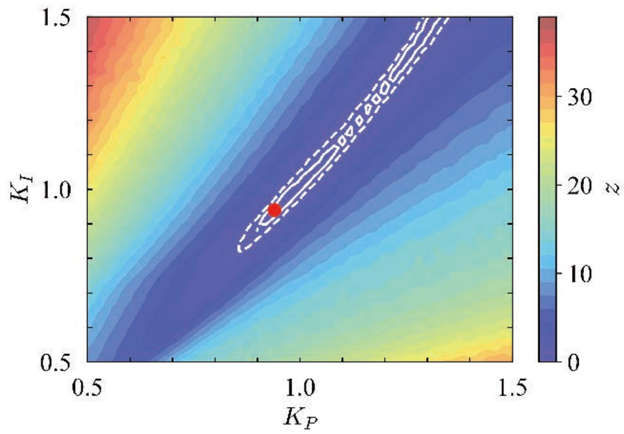


Figure 4 Distribution of the deviation index $z$ for the standard unit (See Eq. (3)).

## 3. PI GAIN TUNING METHOD

The procedure of PI gain tuning follows the next two phases: learning for the standard unit and tuning for test units. For the formulation, the explanatory and objective variables ($\boldsymbol{x}$, $\boldsymbol{y}$) are defined as follows:

$$\boldsymbol{x} = (K_P, K_I), \tag{4}$$

$$\boldsymbol{y} = (T_r, Q_{os}), \tag{5}$$

and $\boldsymbol{x}_0 = (K_{P0}, K_{I0})$, $\boldsymbol{y}_0 = (T_{r0}, Q_{os0})$.

### 3.1 Learning for standard unit

From the collected 2,500 data of ($\boldsymbol{x}$, $\boldsymbol{y}$) for the standard unit, the probability density distribution is modeled as the form of the GMM:

$$p(\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{n_G} \pi_i N\left([\boldsymbol{x},\boldsymbol{y}] \middle| [\boldsymbol{\mu}_{x,i}, \boldsymbol{\mu}_{y,i}], \begin{bmatrix} \Sigma_{xx,i} & \Sigma_{yx,i} \\ \Sigma_{xy,i} & \Sigma_{yy,i} \end{bmatrix}\right). \tag{6}$$

Here, $\pi_i$ is the weight of the $i$ -th Gaussian, $\boldsymbol{\mu}_{x,i}$, $\boldsymbol{\mu}_{y,i}$ are the mean vectors of $\boldsymbol{x}$, $\boldsymbol{y}$, and $\Sigma_{xx,i}$, $\Sigma_{xy,i}$, $\Sigma_{yx,i}$, $\Sigma_{yy,i}$ are their variance-covariance matrices, respectively, which are determined by the expectation-maximization method. The number of Gaussian $n_G$ is determined to minimize the square error without an overfitting. In this case, we set $n_G = 10$.

### 3.2 PI gain tuning for test units

A step response waveform is acquired for each test unit by setting the initial gain at $\boldsymbol{x} = \boldsymbol{x}_0$. If the p($\boldsymbol{x}$, $\boldsymbol{y}$) for the considering test unit is almost the same as the one for the standard unit, the resulting $\boldsymbol{y}$ is supposed to meet the specifications immediately. If the resulting $\boldsymbol{y}$ fails to meet the specifications, the direct inverse analysis of GMM with respect to the $\boldsymbol{y}$ is applied to get the predictive gains $\hat{\boldsymbol{x}} = (\hat{K}_P, \hat{K}_I)$, which is expected to give the same $\boldsymbol{y}$ for the standard unit. The probability density of $\boldsymbol{x}$ under given $\boldsymbol{y}$ can be calculated as,

$$p(\boldsymbol{x}|\boldsymbol{y}) = \sum_{i=1}^{n_G} \omega_{y,i}\, p(\boldsymbol{x}|\boldsymbol{y}, \boldsymbol{\mu}_{y,i}, \Sigma_{yy,i}), \tag{7}$$

where $p\,(\boldsymbol{x}|\boldsymbol{y}, \boldsymbol{\mu}_{y,i}, \Sigma_{yy,i})$ is the probability density distribution of $\boldsymbol{x}$ in the $i$ -th Gaussian in Eq. (6) under $\boldsymbol{\mu}_{y,i}$, $\Sigma_{yy,i}$ (yy,i) and given $\boldsymbol{y}$, and $\omega_{y,i}$ is the weight calculated as,

$$\omega_{y,i} = \frac{\pi_i p(\boldsymbol{y}|\boldsymbol{\mu}_{y,i}, \Sigma_{yy,i})}{\sum_{j=1}^{n_G} \pi_j p(\boldsymbol{y}|\boldsymbol{\mu}_{y,j}, \Sigma_{yy,j})}. \tag{8}$$

The mean vector of $\boldsymbol{x}$ in the i-th Gaussian under given $\boldsymbol{y}$, is calculated as,

$$\boldsymbol{m}_i(y) = \boldsymbol{\mu}_{x,i} + (\boldsymbol{y} - \boldsymbol{\mu}_{y,i})\Sigma_{yy,i}^{-1}\Sigma_{yx,i}^{-1}. \tag{9}$$

Assuming that $z$ -distribution for the test unit can be approximately overlapped by simply shifting the distribution for the standard unit in $\boldsymbol{x}$-direction, the newly defined $\boldsymbol{x}$ as,

$$\boldsymbol{x} = \boldsymbol{x}_0 + \Delta\boldsymbol{x}, \tag{10}$$

is expected to give $\boldsymbol{y}$ close to $\boldsymbol{y}_0$ for the test unit. Here, the shift vector is

$$\Delta\boldsymbol{x} = (K_{P0} - \hat{K}_P, K_{I0} - \hat{K}_I). \tag{11}$$

If the new $\boldsymbol{x}$ fails again to give $\boldsymbol{y}$ meeting the specifications, the same cycle is repeated unless the number of iterations $n$ reaches a specific maximum value $n_{max}$. The flow chart of this procedure is summarized in Figure 5.

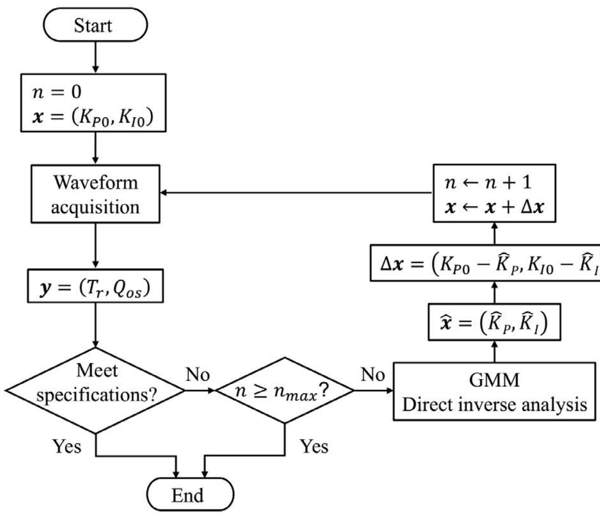


Figure 5 Flow chart of PI tuning for test units.

## 4. RESULTS

We applied the PI gain tuning method for seven test units. The results are listed in Table 1. The evaluation indices that meet the specifications were obtained for all the test units with $n \leq 4$ , though we set $n_{max}$=10.

Table 1 Results of PI tuning for test units

| Unit no. | $(K_P, K_I)$ | $T_r$ (ms) | $Q_{os}$ (%) | $n$ |
|---|---|---|---|---|
| 1 | (0.91, 0.88) | 82 | 0.214 | 2 |
| 2 | (0.94, 0.94) | 86 | −0.061 | 0 |
| 3 | (0.97, 0.91) | 86 | −0.165 | 1 |
| 4 | (0.90, 0.86) | 84 | −0.161 | 4 |
| 5 | (0.90, 0.85) | 88 | −0.228 | 4 |
| 6 | (0.93, 0.89) | 84 | −0.118 | 1 |
| 7 | (0.99, 0.98) | 84 | −0.174 | 1 |

The required iterations were different depending on individual units as $n$= 0-4. To consider the difference, we investigated $z$-distributions of units with the minimum (no.2) and maximum (no.4) iterations, as shown in Figure 6. Both units indicated similar distributions to the standard unit's distribution having the necessary and sufficient regions around $K_P \approx K_I$. Thus, the assumption that a test unit's $z$-distribution can be approximately overlapped by simply shifting the standard unit's one in $x$-direction was appropriate. As the location and area of necessary and sufficient region in no. 4 differed more from those of the standard unit's distribution compared to no. 2, no. 4 unit required more iterations than no. 2. However, the differences were small enough to be compensated by repeating the cycle of the direct inverse analysis and $x$-direction shift. As the PI gains likely to realize the target waveform are set repeatedly, we can achieve the target with a few iterations.

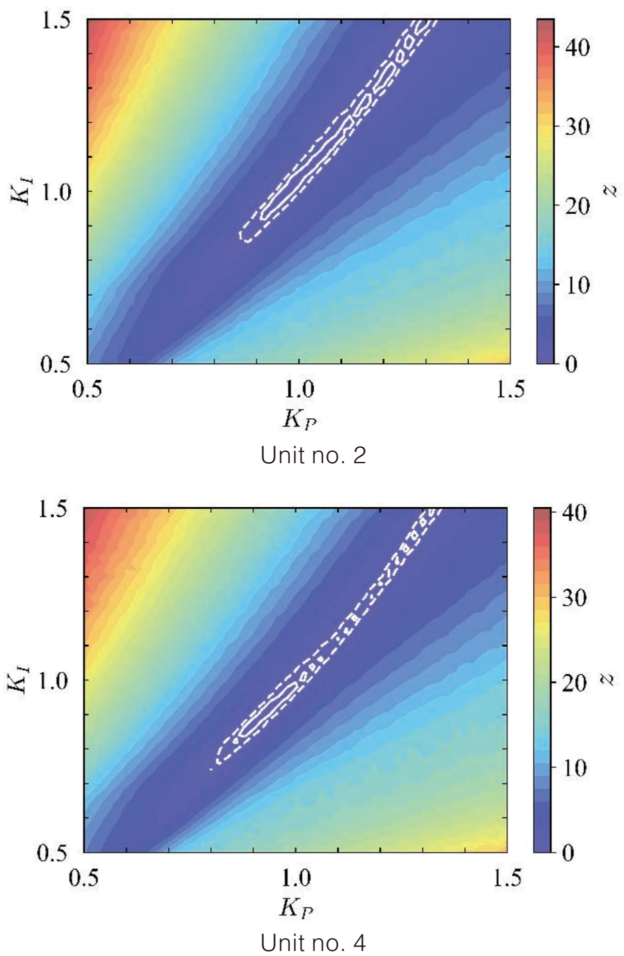


Figure 6 Comparison of distributions of the deviation index z (See Eq. (3))

## 5. CONCLUSION

The method for PI gain tuning using the GMM and the direct inverse analysis applicable to pressure-based MFCs' production are proposed. We applied the method to seven test units. The result showed that the gains of all the test units were tuned within only a few iterations. As this method can efficiently find the optimal gains that are located at a narrow region in the $K_P$ - $K_I$ plane, it is promising for the mass production of the pressure-based MFC whose control target is significantly complicated. We believe this method can simplify the manufacturing process for the complicated pressure-based MFC and contribute its stable delivery.

* Editorial note: This content is based on HORIBA's investigation at the year of publication unless otherwise stated.

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