Flow and Storage of Powders

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Happiness Equals Reality Minus Expectations

Flow and Storage of Powders

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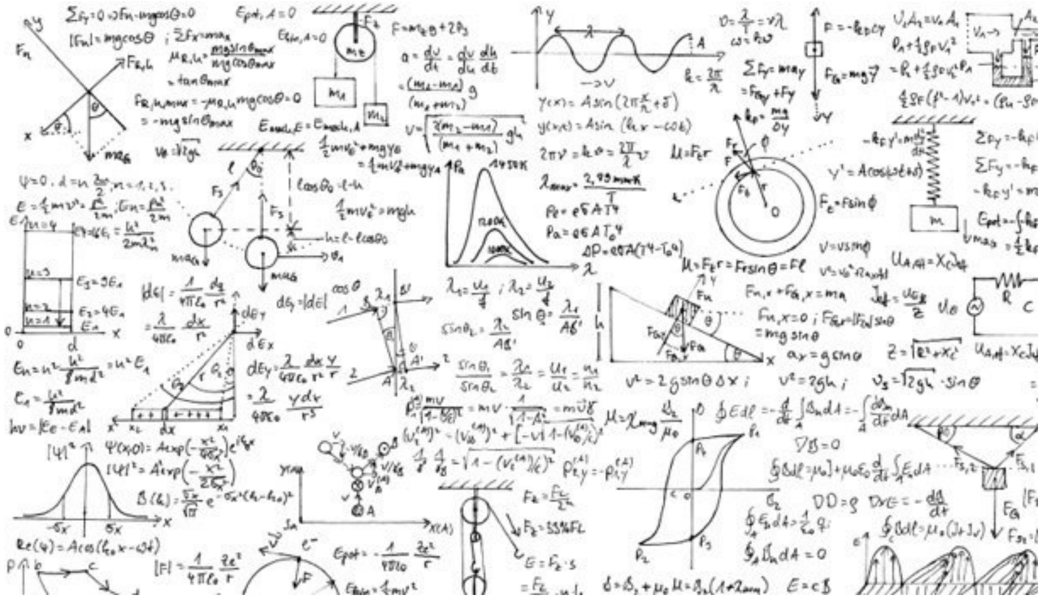

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Flow and Storage of Powders

Goals

- Understand *fundamental* powder flow properties
- Predict flow behavior of powders
- Know how to design/modify equipment for reliable powder handling

Goals

GM&A

Goals

Definitions

Bulk solid

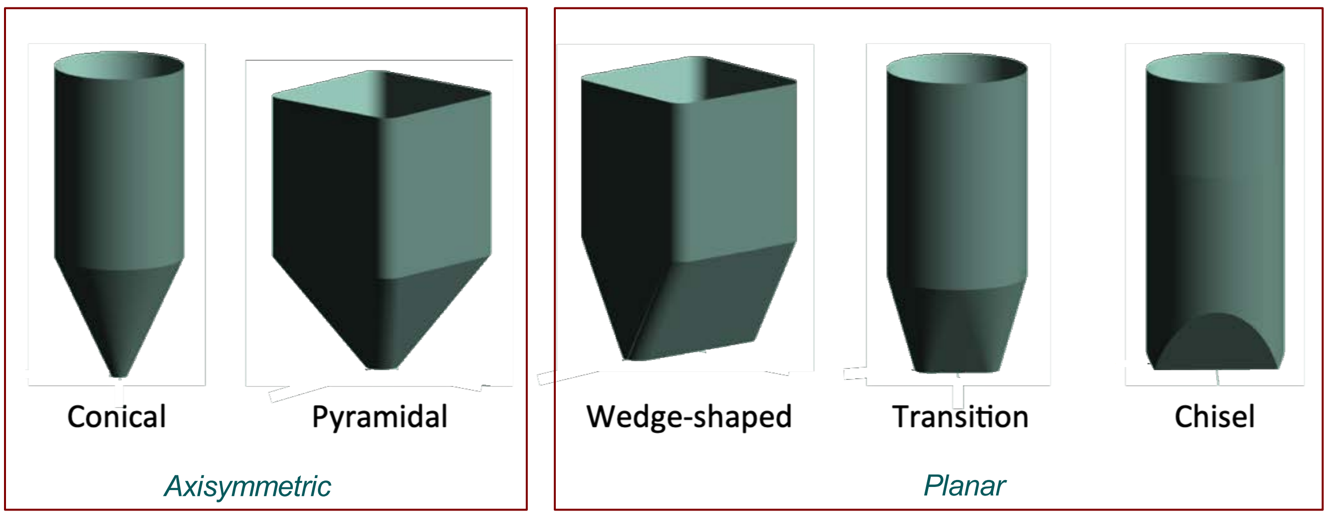
"Powder"

Definitions

Biomass

Definitions

Hopper, bin, or silo



Definitions

- Normal stress
- Shear stress
- Major principal stress

Classification of liquids

Newtonian	Stress is proportional to strain rate
Viscoelastic	Combination of viscous and elastic behaviors
Rheopectic	Viscosity increases over time
Thixotropic	Viscosity decreases over time
Pseudoplastic	Viscosity increases with increased stress
Dilatant	Viscosity decreases with increased stress

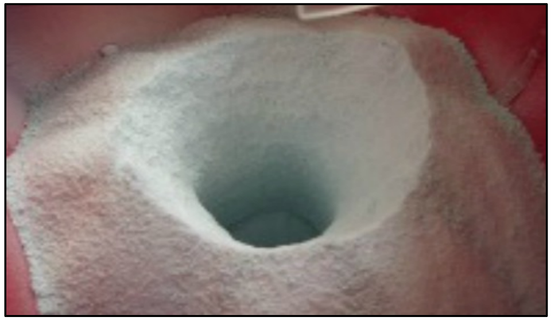
Classification of bulk solids

Neurotic	move awkwardly - poor flowability - sticky or tacky
Sadistic	attack their surroundings - abrasive - explosive
Masochistic	suffer from their surroundings - friable - degradable
Schizophrenic	change their behavior pattern - hygroscopic - electrostatic

Woodcock and Mason, 1987

Flow problems

- Ratholing



Flow problems

- Ratholing
- Arching/bridging

Bridging

Flow problems

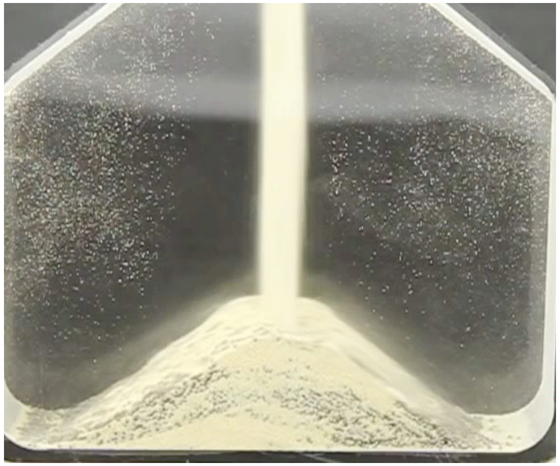
- Ratholing
- Arching
- Caking

Flow problems

- Ratholing
- Arching
- Caking
- Flooding

Flow problems

- Ratholing
- Arching
- Caking
- Flooding
- Segregation



Flow problems

- Ratholing
- Arching
- Caking
- Flooding
- Segregation
- Limiting discharge rate

GM&A

Flow problems

- Ratholing
- Arching
- Caking
- Flooding
- Segregation
- Limiting discharge rate
- Structural

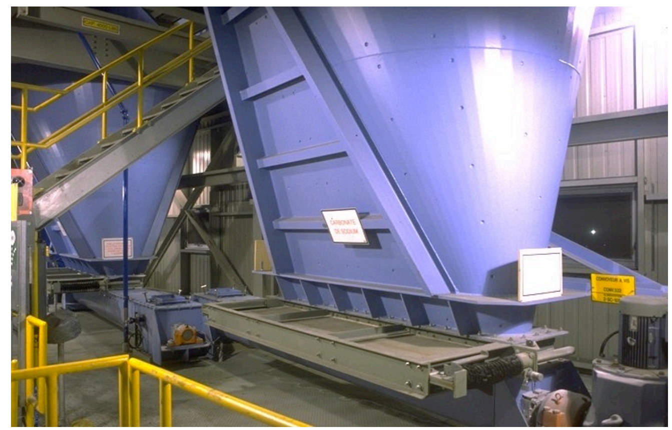
Solids flow patterns

- Funnel flow
- Mass flow

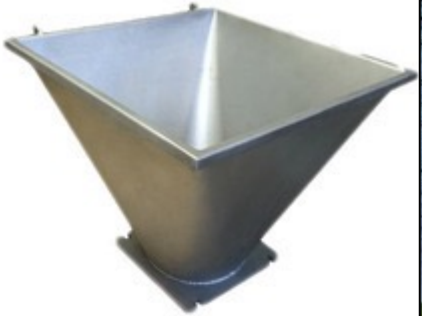
Funnel flow

Mass flow

Mass flow



Funnel flow



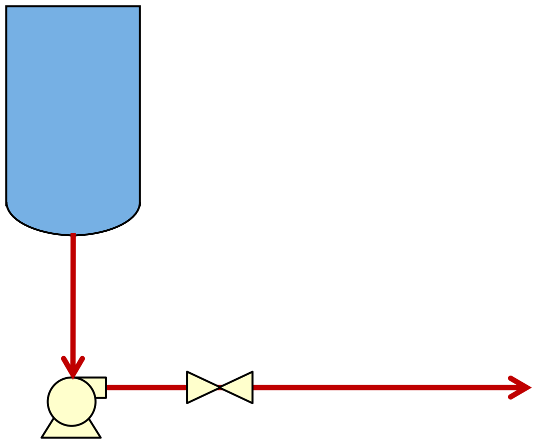
No flow

Preventing flow problems

- Ensure properly sized outlet
- Design/modify to allow mass flow
- Need powder flow properties

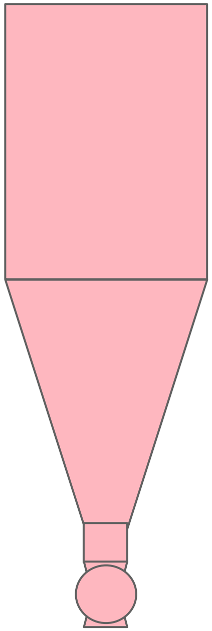
Fundamental properties

- Viscosity
- Density
- Vapor pressure



Fundamental properties

- Cohesive strength
- Internal friction
- Compressibility
- Wall friction
- Permeability



Andrew Jenike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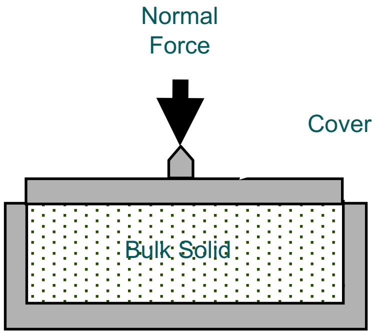
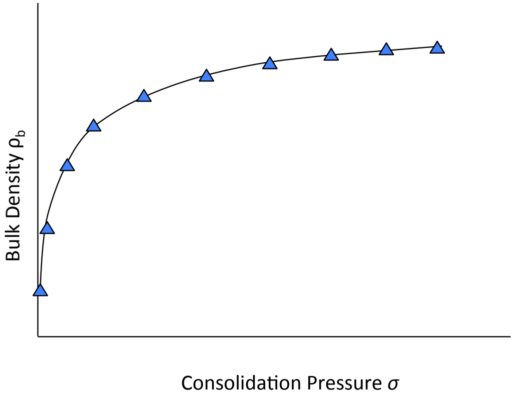
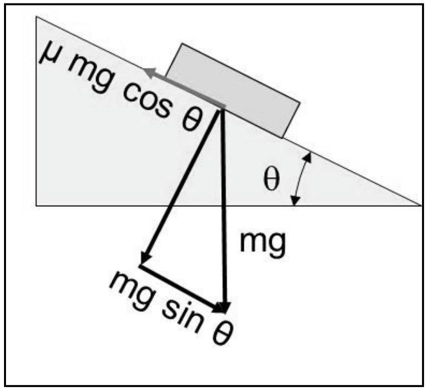
Compressibility

Compressibility





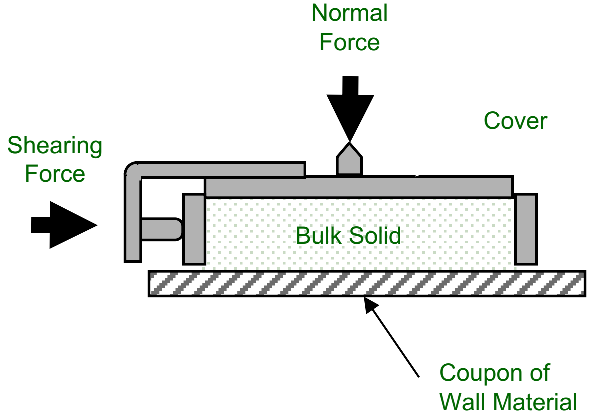
Wall friction

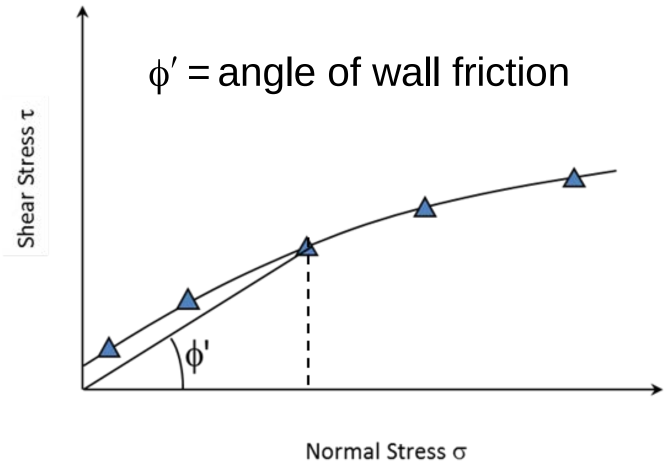


$$mg \sin\theta = \mu\, mg \cos\theta$$

$$\mu = \tan\theta$$

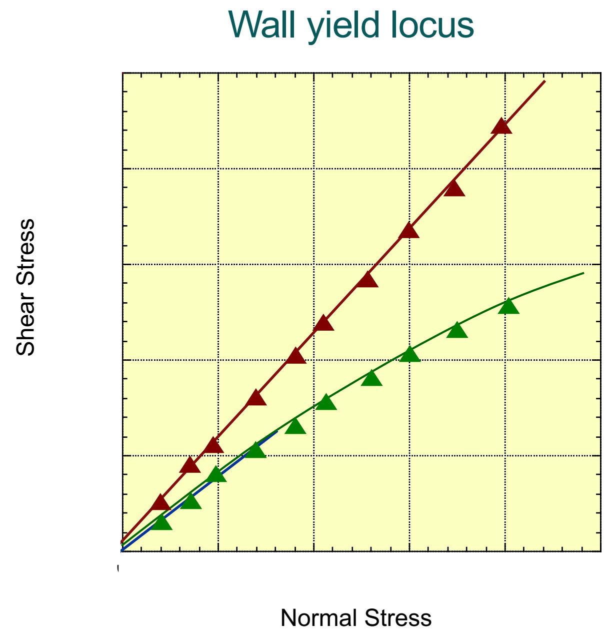
Wall friction



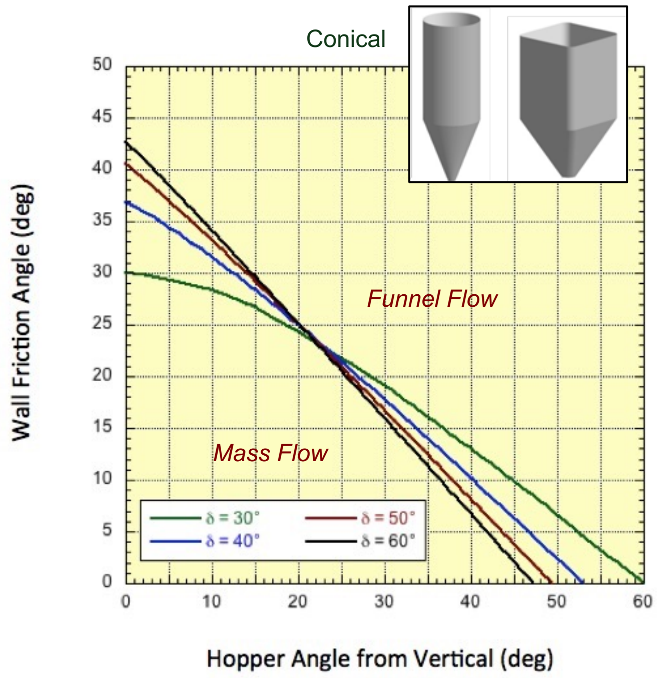


$$\tan\phi' = \frac{\text{Shear Stress}}{\text{Normal Stress}} = \mu$$

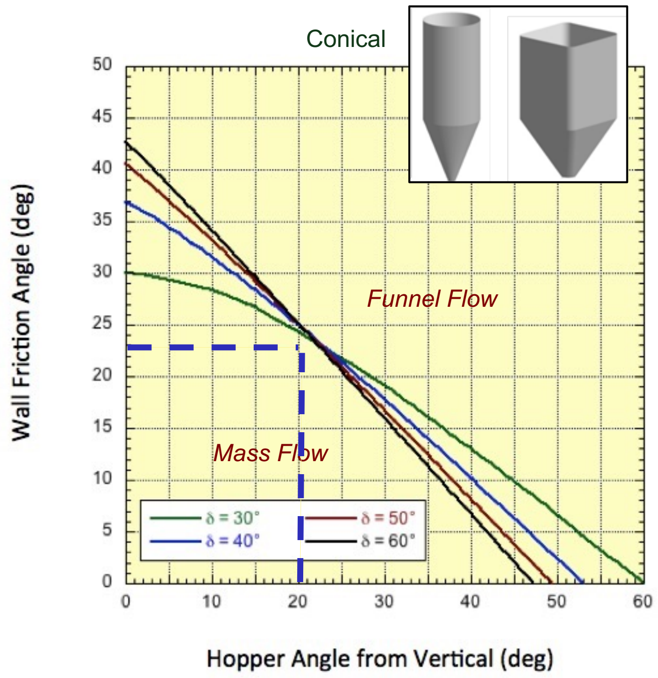
Wall friction



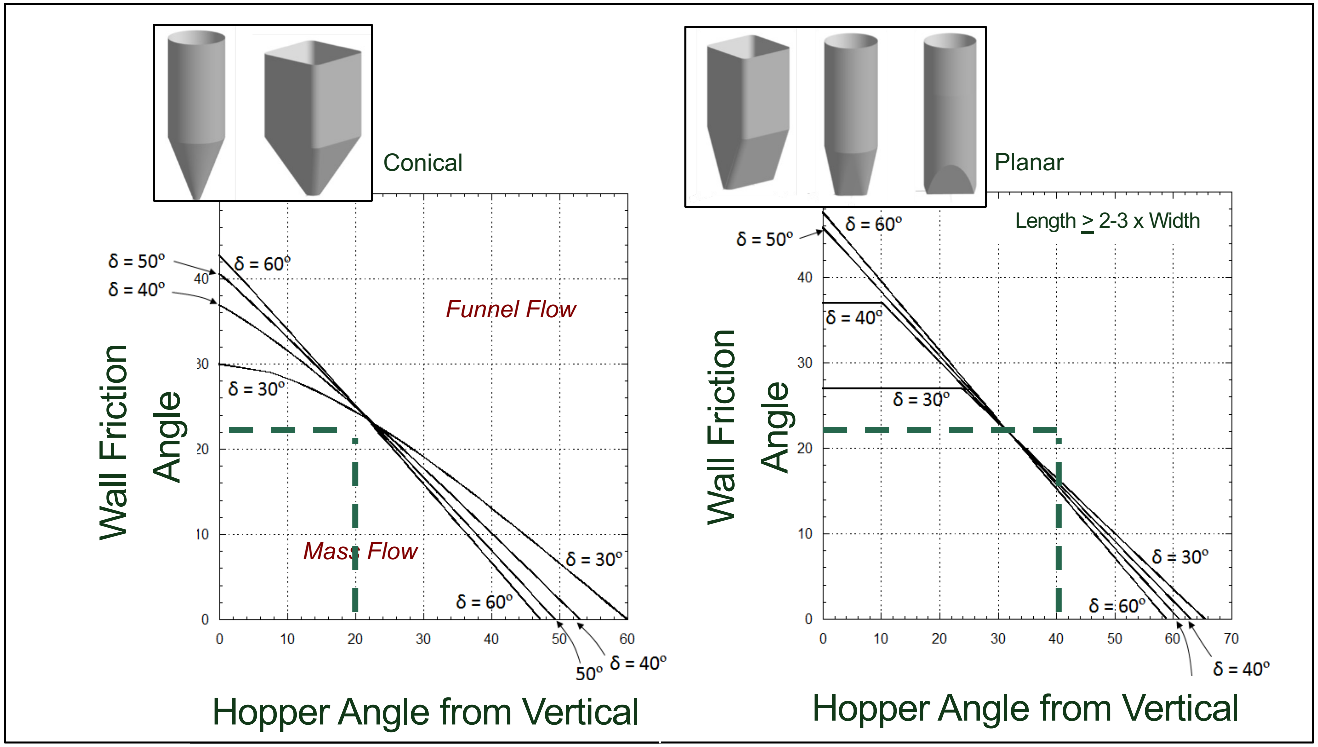
Mass flow hopper angle



Mass flow hopper angle

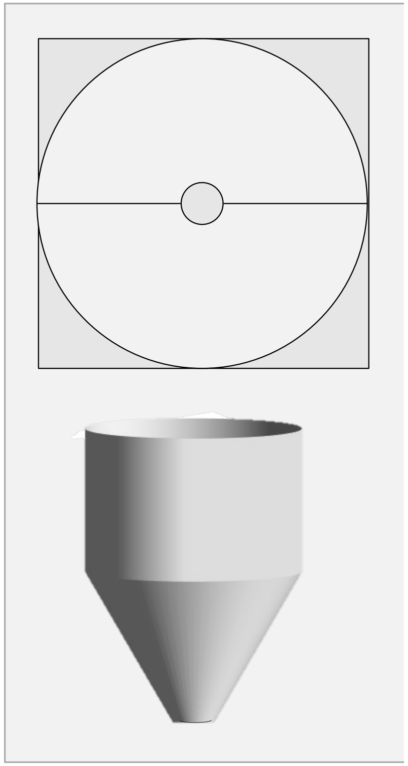


Mass flow hopper angles



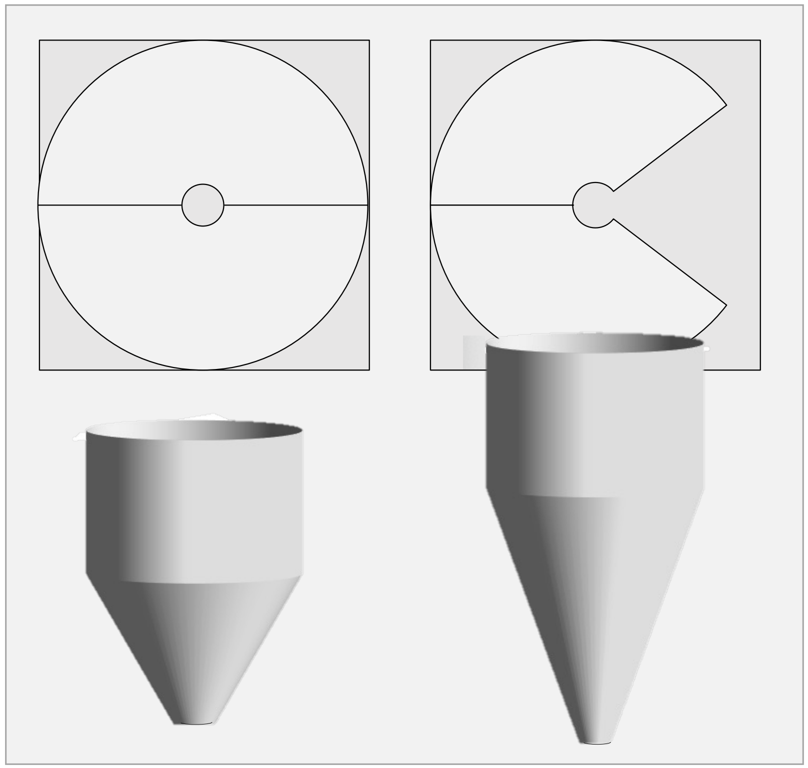
From Jenike Bulletin 123

Mass flow hopper angle



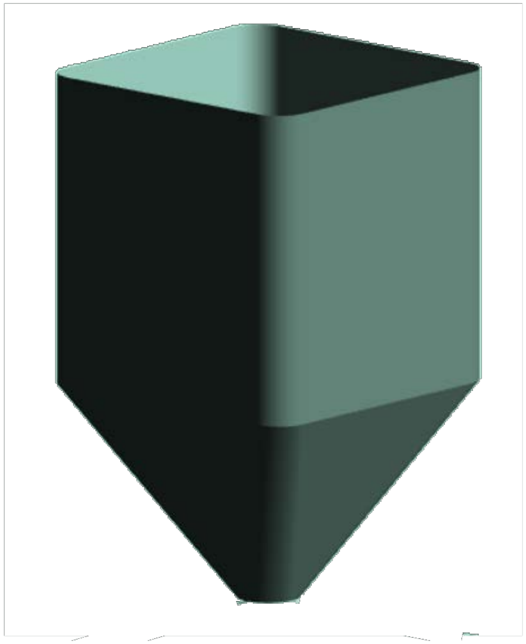
30˚

Mass flow hopper angle

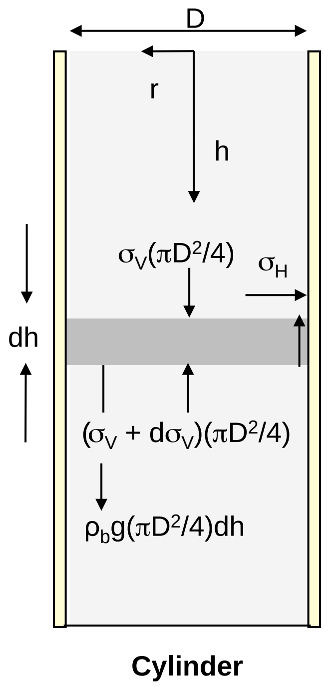


30°　　　　　　20°

Valley angles



Pressure distribution



$$(\sigma_V + d\sigma_V)\frac{\pi D^2}{4} + \sigma_H \mu \pi D\, dh = \sigma_V \frac{\pi D^2}{4} + \rho_b g \frac{\pi D^2}{4} dh$$

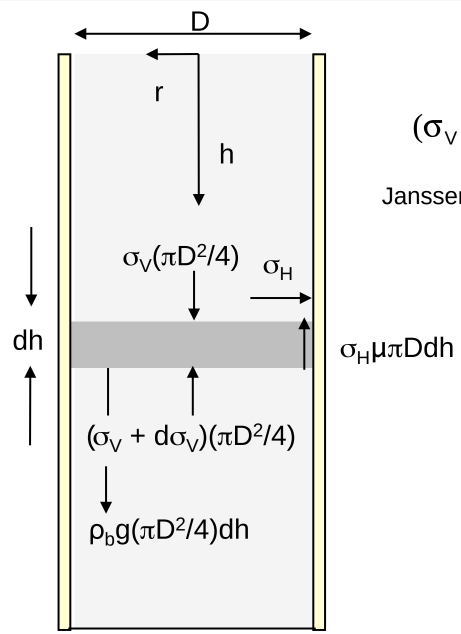
Force from powder below

Force due to wall friction

Force from powder above

Force due to gravity

Pressure distribution



Cylinder

$$(\sigma_V + d\sigma_V)\frac{\pi D^2}{4} + \sigma_H \mu \pi D\, dh = \sigma_V \frac{\pi D^2}{4} + \rho_b g \frac{\pi D^2}{4} dh$$

Janssen coefficient

$$k = \frac{\sigma_H}{\sigma_V} \cong 0.4 - 0.6$$

$$\frac{D}{4}\frac{d\sigma_V}{dh} + k\mu\sigma_V = \frac{D}{4}\rho_b g$$

1st order linear differential equation

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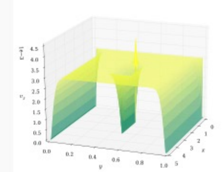
Integrating factor

From Wikipedia, the free encyclopedia
(Redirected from Integration factor)

In mathematics, an **integrating factor** is a function that is chosen to facilitate the solving of a given equation involving differentials. It is commonly used to solve ordinary differential equations, but is also used within multivariable calculus when multiplying through by an integrating factor allows an inexact differential to be made into an exact differential (which can then be integrated to give a scalar field). This is especially useful in thermodynamics where temperature becomes the integrating factor that makes entropy an exact differential.

Differential equations



Navier–Stokes differential equations used to simulate airflow around an obstruction.

Scope	[show]
Classification	
Types	[show]
Relation to processes	[show]
Solution	
General topics	[show]
Solution methods	[show]
People	[show]

V · T · E

Contents [hide]

Use in solving first order linear ordinary differential equations [edit]

Integrating factors are useful for solving ordinary differential equations that can be expressed in the form

$$y' + P(x)y = Q(x)$$

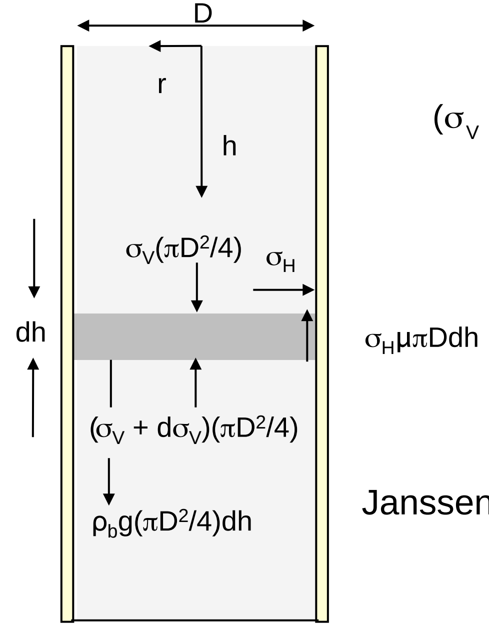
The basic idea is to find some function $M(x)$, called the "integrating factor," which we can multiply through our DE in order to bring the left-hand side under a common derivative. For the canonical first-order, linear differential equation shown above, our integrating factor is chosen to be

$$M(x) = e^{\int_{s_0}^{x} P(s)ds}$$

In order to derive this, let $M(x)$ be the integrating factor of a first order, linear differential equation such that multiplication by $M(x)$ transforms a partial derivative into a total derivative, then:

$$(1) \qquad M(x)\underbrace{(y' + P(x)y)}_{\text{partial derivative}}$$

Pressure distribution



Cylinder

$$(\sigma_V + d\sigma_V)\frac{\pi D^2}{4} + \sigma_H \mu \pi D\, dh = \sigma_V \frac{\pi D^2}{4} + \rho_b g \frac{\pi D^2}{4} dh$$

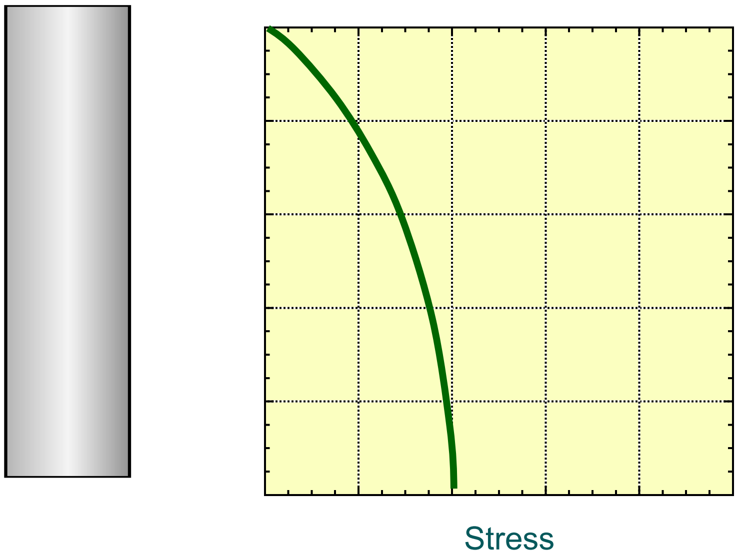
$$k = \frac{\sigma_H}{\sigma_V} \cong 0.4$$

$$\frac{D}{4}\frac{d\sigma_V}{dh} + k\mu\sigma_V \quad \frac{D}{4}\rho_b g$$

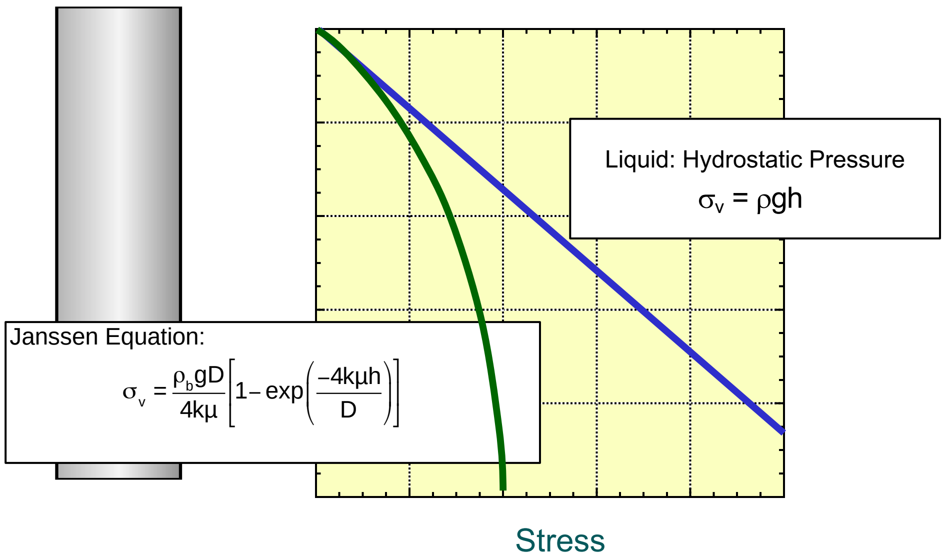
Janssen Equation:

$$\sigma_v = \frac{\rho_b g D}{4k\mu}\left[1 - \exp\left(\frac{-4k\mu h}{D}\right)\right]$$

Pressure distribution

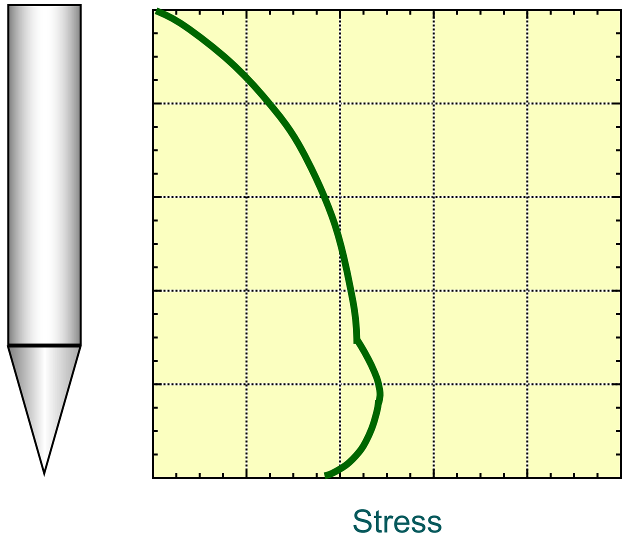


Pressure distribution



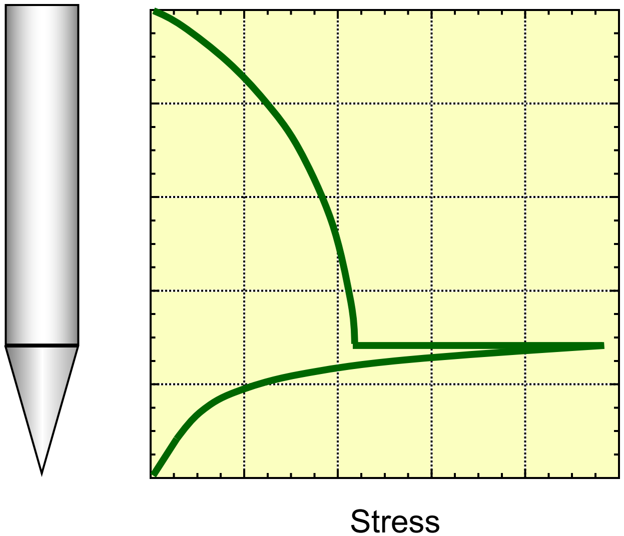
Pressure distribution

Initial fill

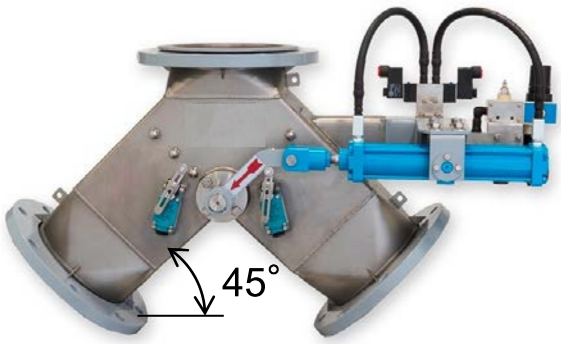


Pressure distribution

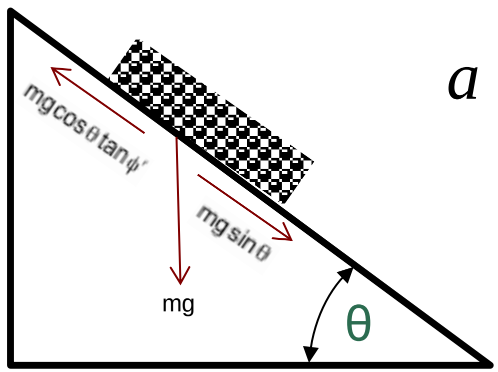
After discharge (mass flow)



Transfer chutes and diverter valves



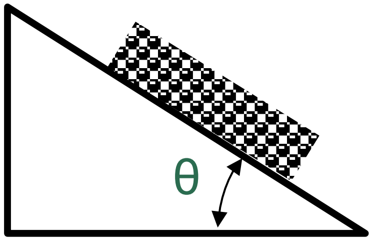
Transfer chutes



$$a = g(\sin\theta - \cos\theta\tan\phi')$$
$$= g\cos\theta(\tan\theta - \tan\phi')$$

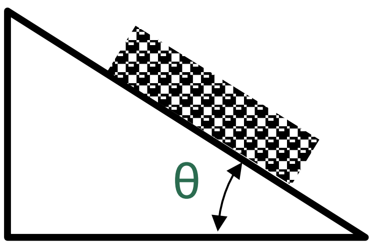
For dependable flow, $\theta' = \phi' + 5^\circ$

Transfer chutes



$$a = g\cos\theta(\tan\theta - \tan\phi')$$

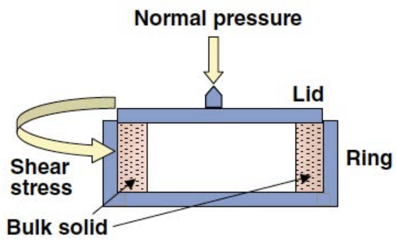
Transfer chutes

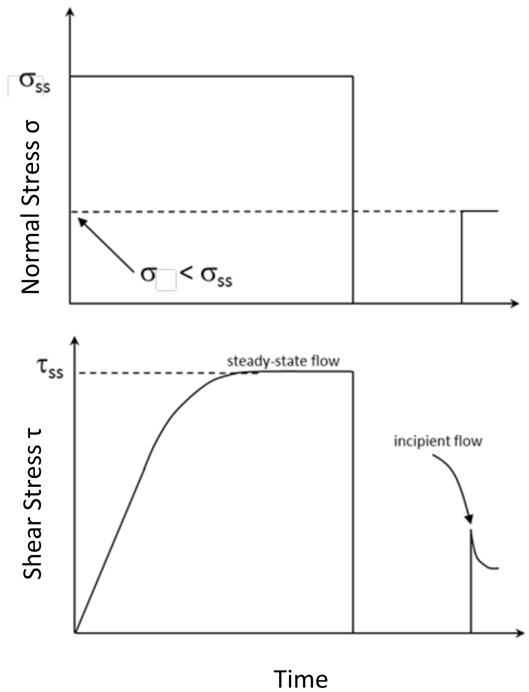


$$a = g\cos\theta(\tan\theta - \tan\phi')$$

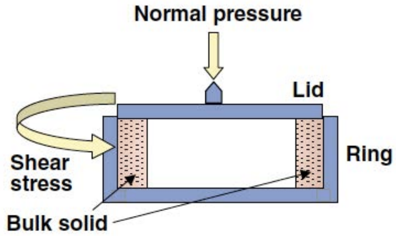
Cohesive strength

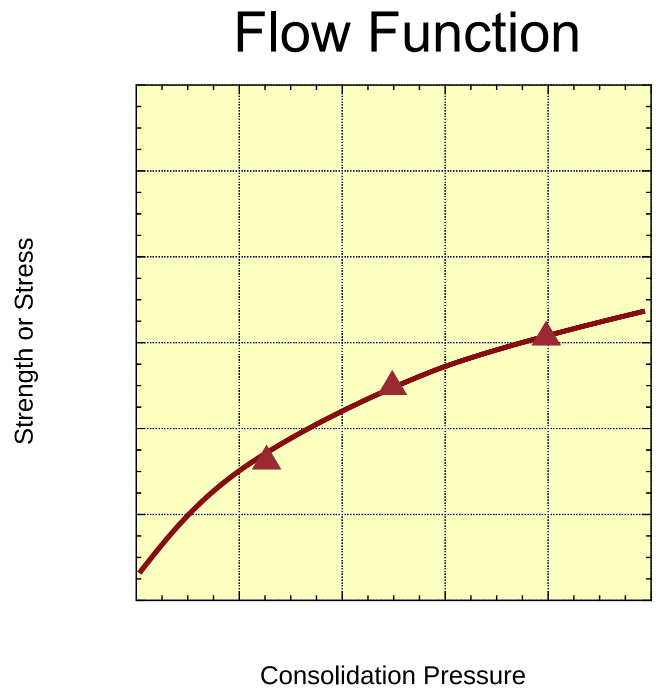
Cohesive strength





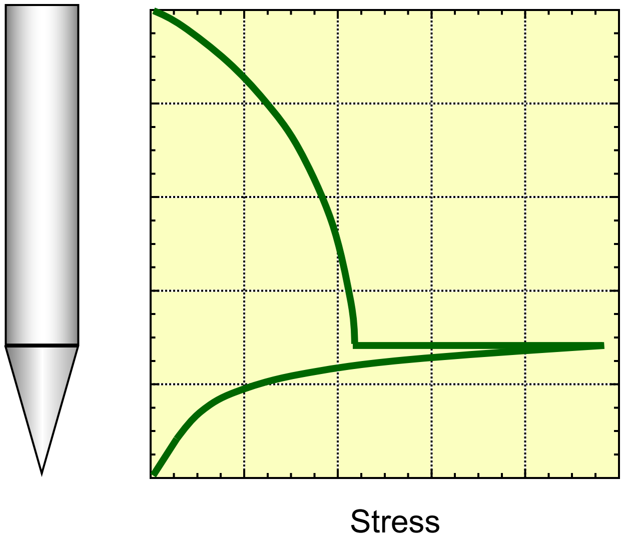
Cohesive strength



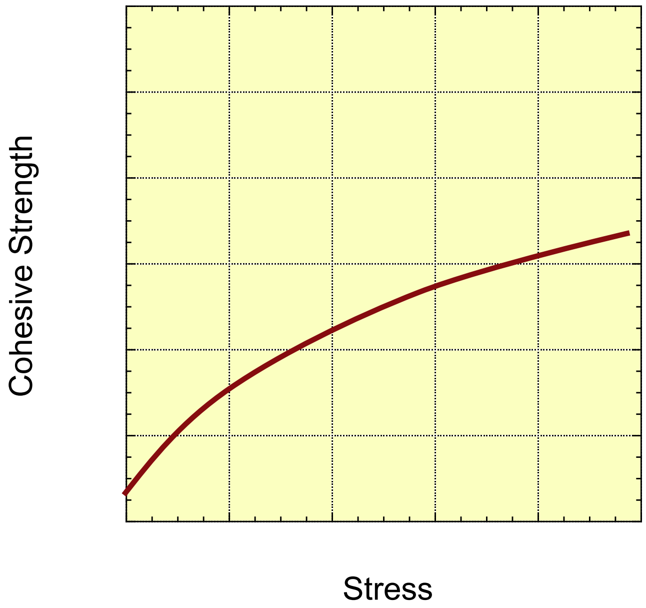


Pressure distribution

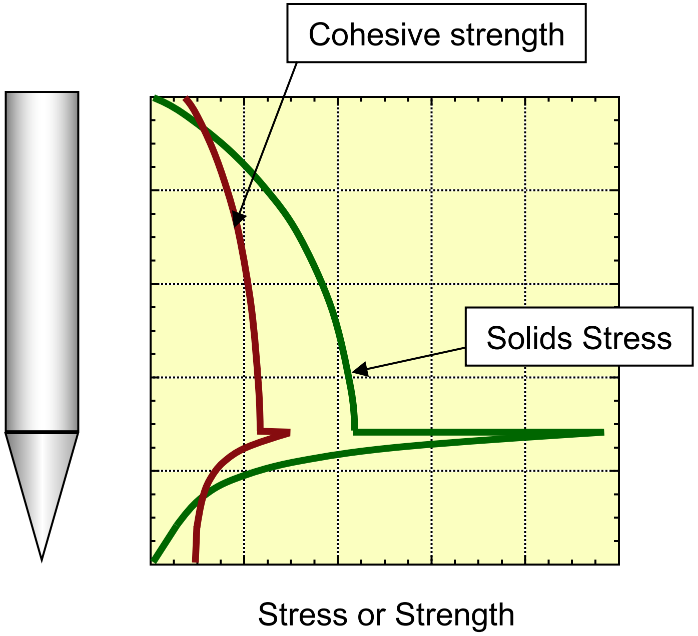
After discharge (mass flow)



Cohesive strength



Cohesive strength



Arch stress

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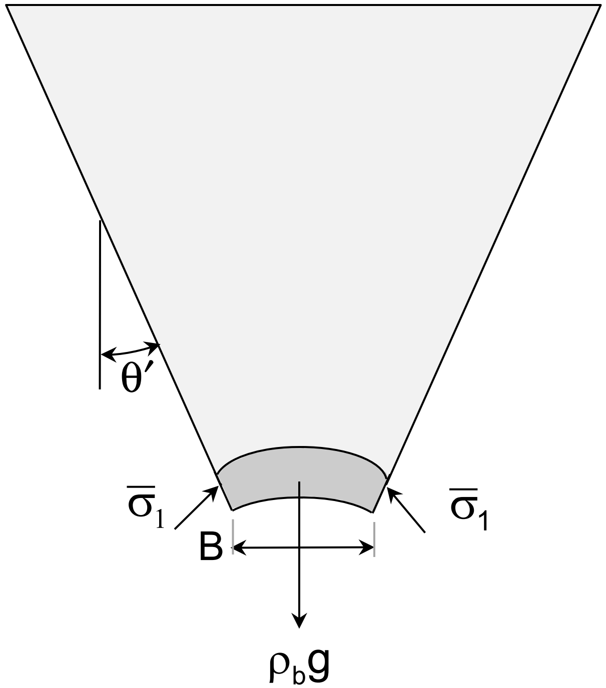
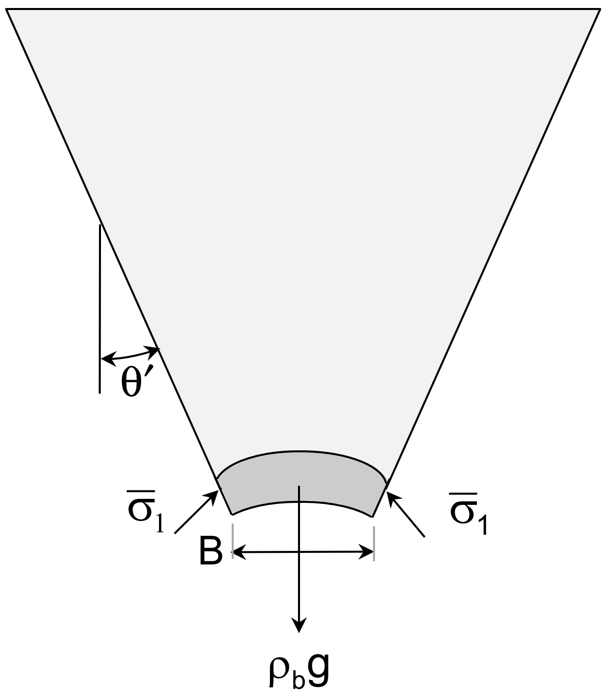
Arch stress



Stress on abutments of arch:

$$\bar{\sigma}_1 \approx \frac{\rho_b g B}{2}$$

Arch stress



Stress on abutments of arch:

$$\bar{\sigma}_1 = \frac{\rho_b g B}{H(\theta')}$$

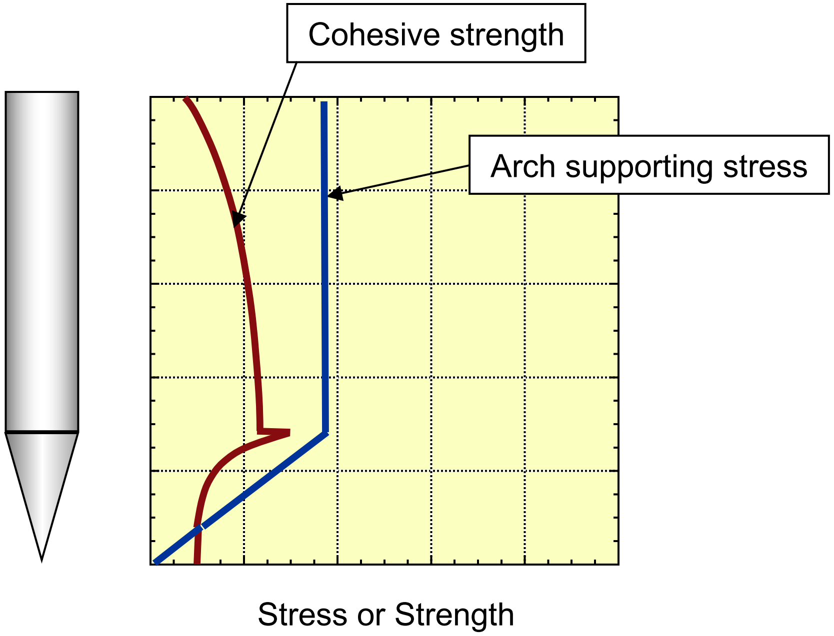
Conical:

$$H(\theta') \approx 2$$

Slot:

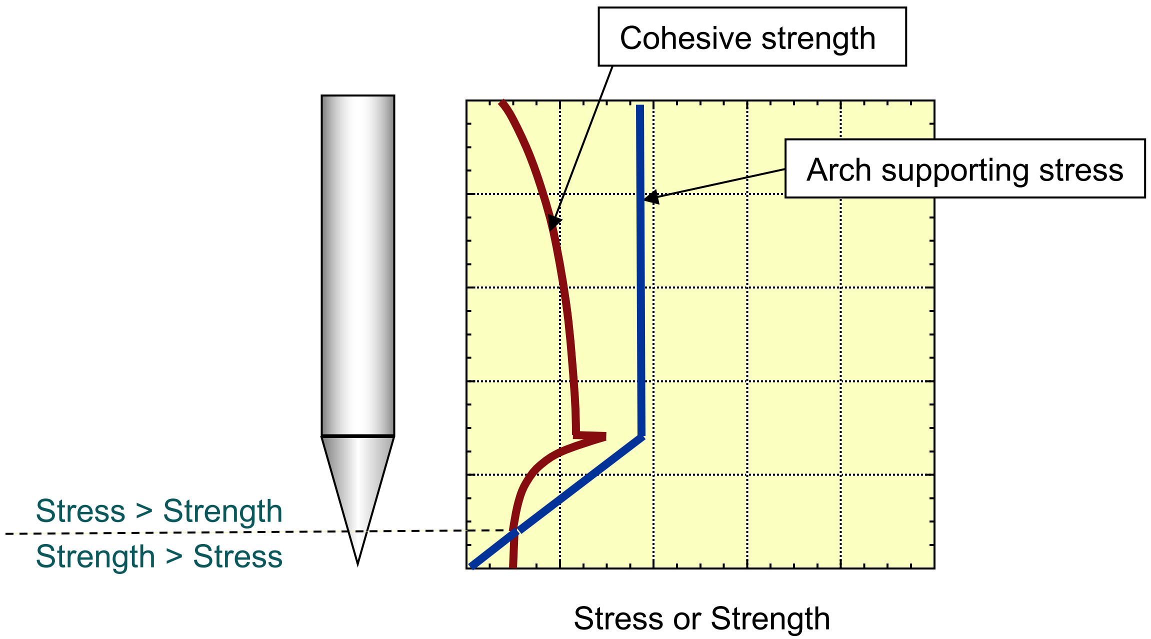
$$H(\theta') \approx 1$$

Arch stress



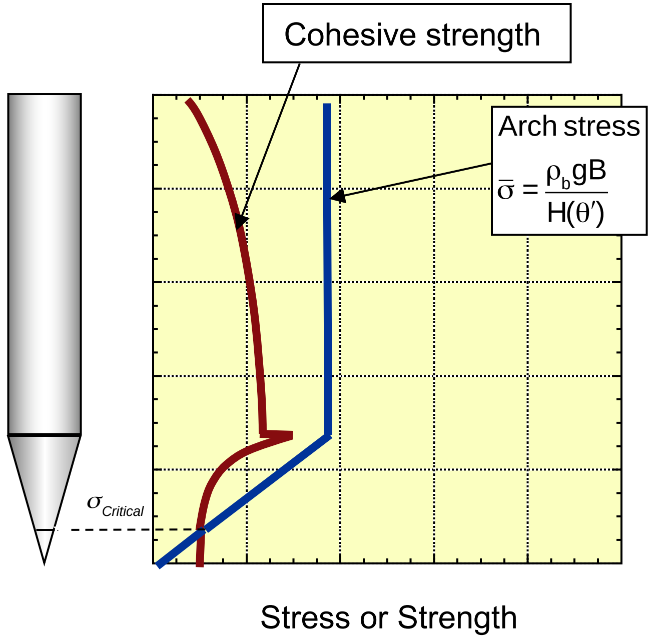
Cohesive strength
Arch supporting stress
Stress or Strength

Arch stress



Cohesive strength
Arch supporting stress
Stress > Strength
Strength > Stress
Stress or Strength

Critical arching diameter



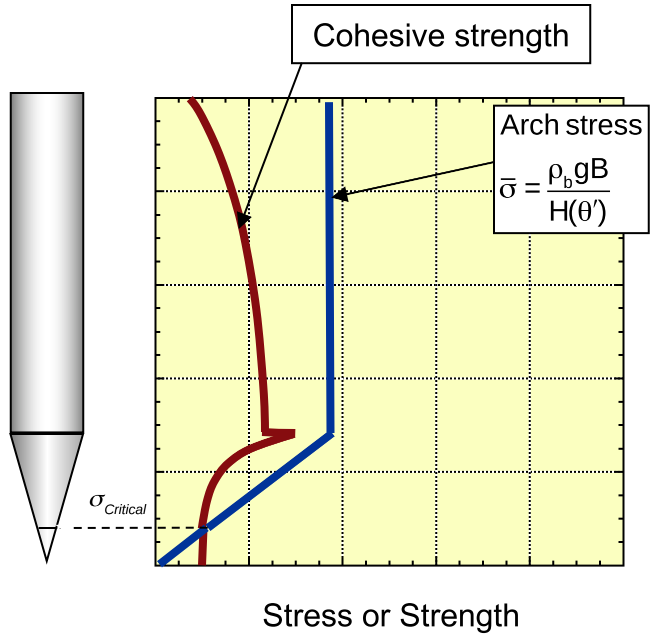
Critical stress
Stress = Strength

Critical arching diameter

$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Critical stress

Stress = Strength

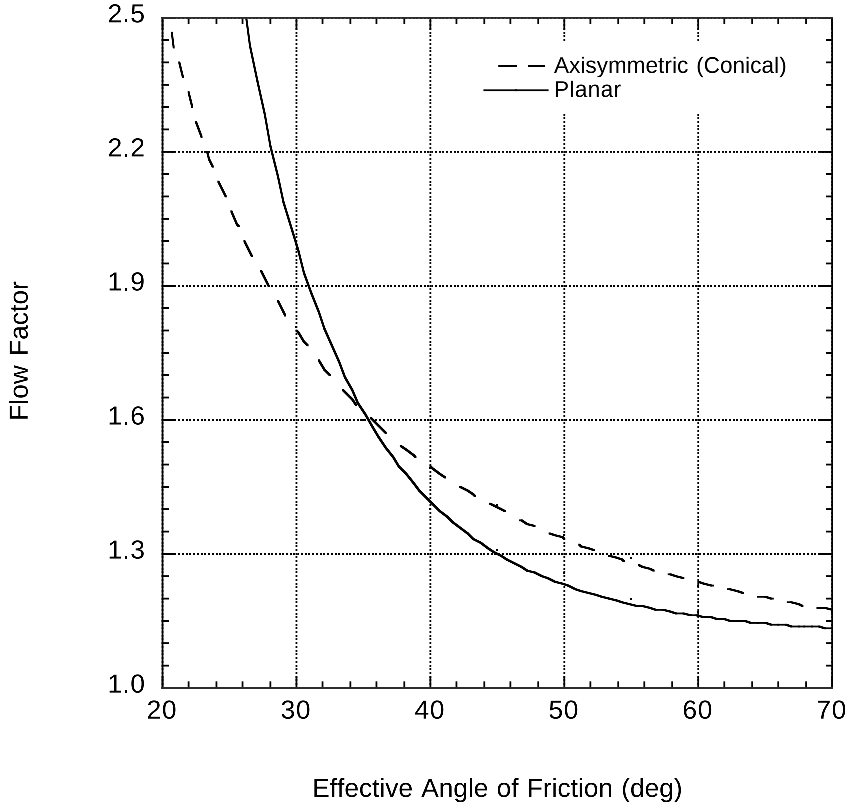


Flow factor

- Flow function (FF)
  - Relationship between consolidation stress and cohesive strength
- Flow factor (ff)
  - Ratio of consolidation stress and arch supporting stress

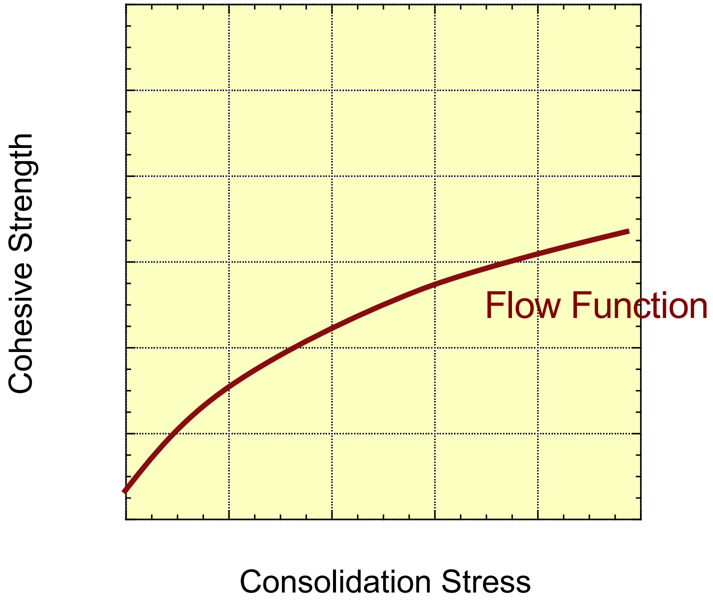
$$ff = \frac{\text{consolidation stress}}{\text{arch stress}}$$

Critical flow factor

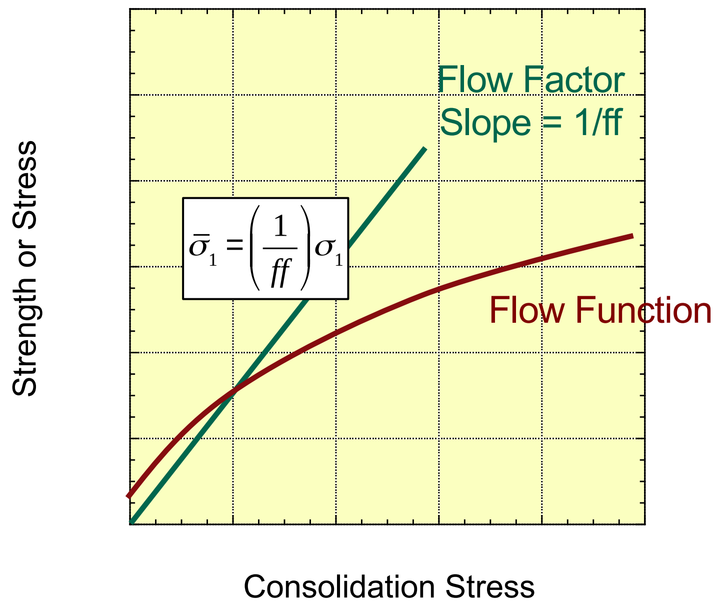


from Jerry Johanson's chapter in McGlinchey, Bulk Solids Characterization, 2005

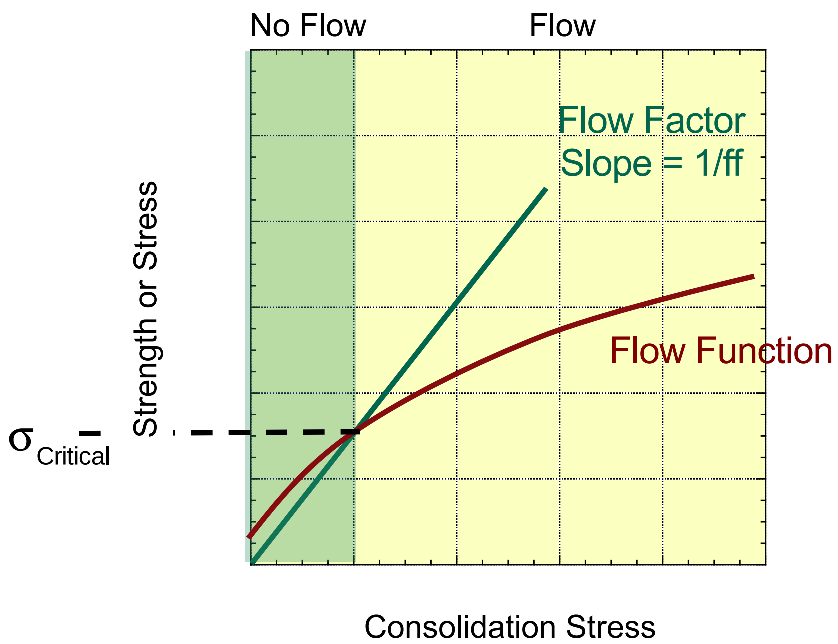
Flow function



Flow factor


Flow Factor
Slope = 1/ff
$\bar{\sigma}_1 = \left(\frac{1}{ff}\right)\sigma_1$
Flow Function
Strength or Stress
Consolidation Stress

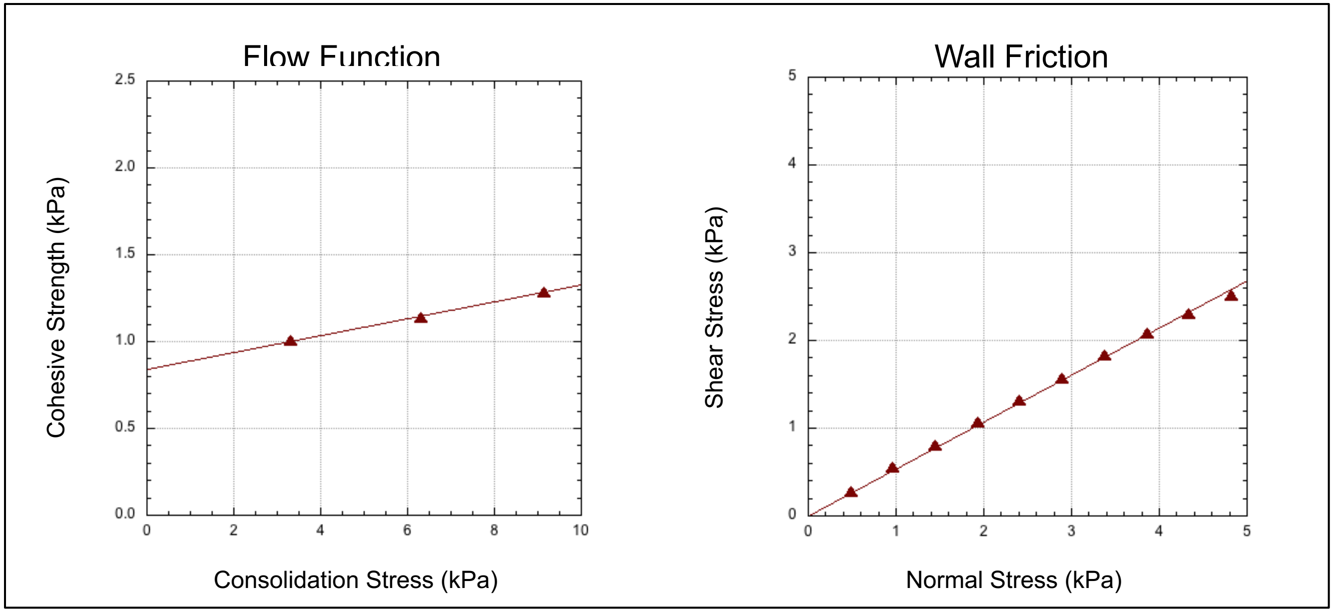
Critical stress



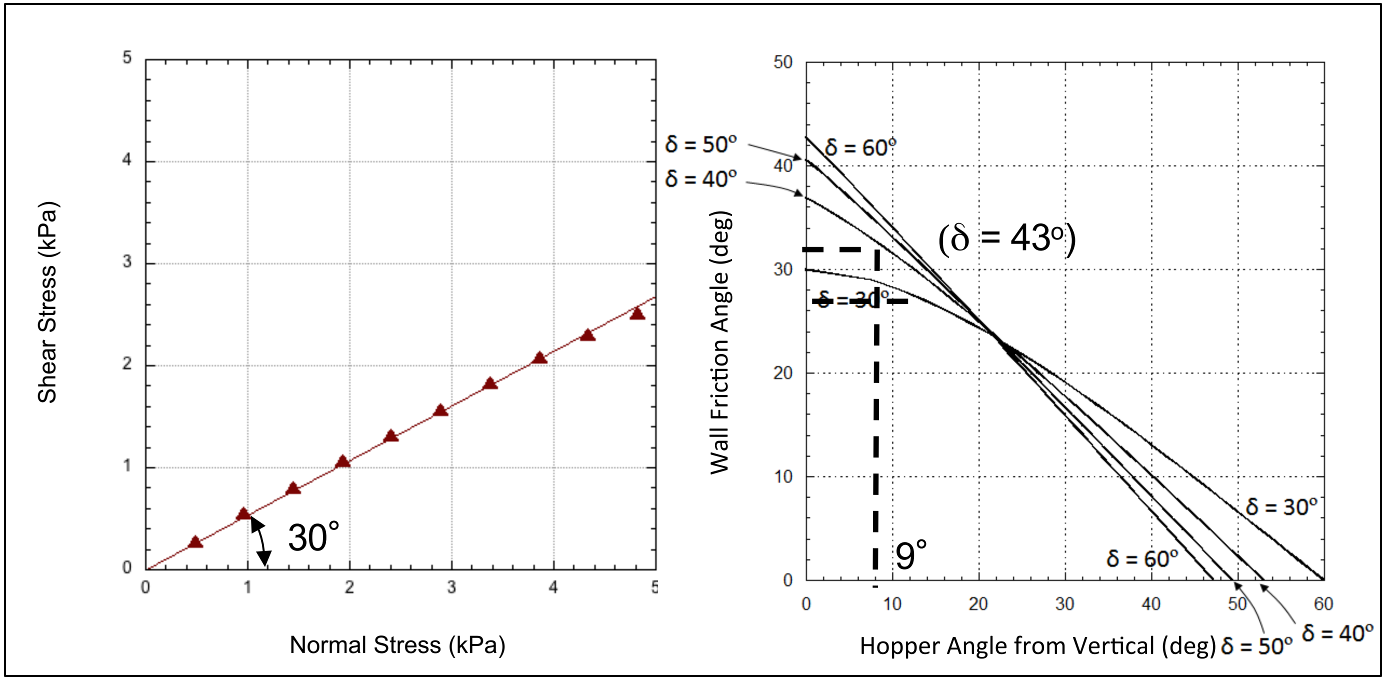
$$B_{min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g}$$

Example

Design a hopper that reliable handles 67/29/4 blend of acetaminophen/Avicel/hydroxypropyl cellulose

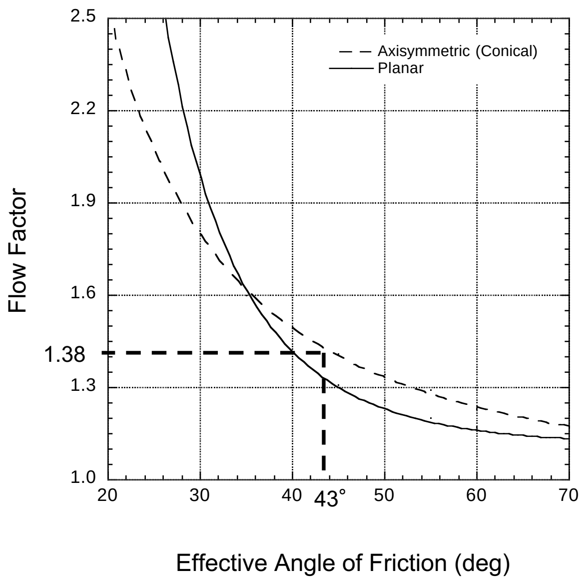


Example

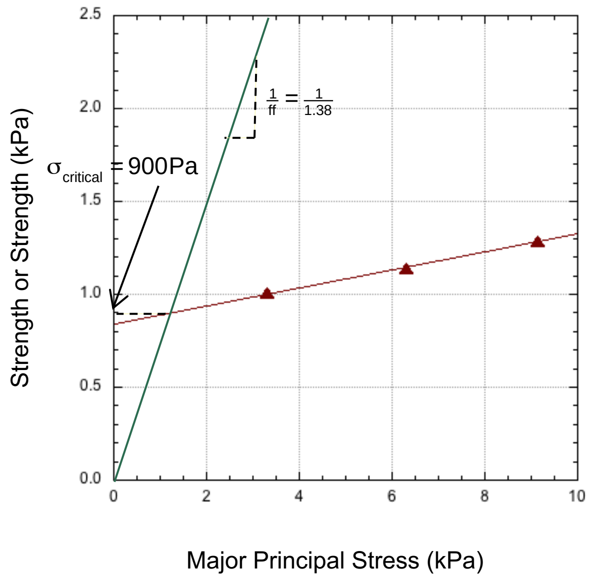


Shear Stress (kPa)
Normal Stress (kPa)
30°
Wall Friction Angle (deg)
Hopper Angle from Vertical (deg)
δ = 60°
δ = 50°
δ = 40°
δ = 30°
(δ = 43°)
9°

Example



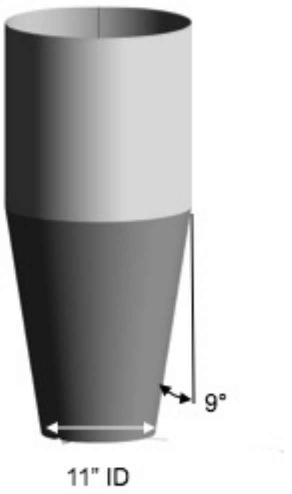
2.5
2.2
1.9
1.6
1.38
1.3
1.0
Flow Factor
Axisymmetric (Conical)
Planar
20
30
40
43°
50
60
70
Effective Angle of Friction (deg)



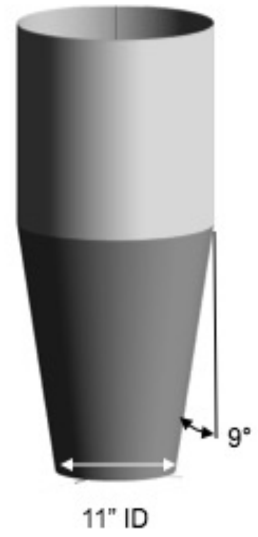
2.5
2.0
1.5
1.0
0.5
0.0
Strength or Strength (kPa)
$\frac{1}{ff} = \frac{1}{1.38}$
$\sigma_{critical}$ = 900Pa
0
2
4
6
8
10
Major Principal Stress (kPa)

Example

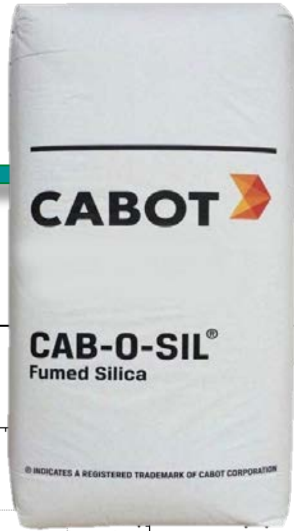
$$B_{\min} = \frac{H(\theta')\sigma_{Critical}}{\rho_b g} = \frac{(2.14)(900)}{(690)(9.8)} = 0.28 \text{ m} = 11 \text{ in.}$$

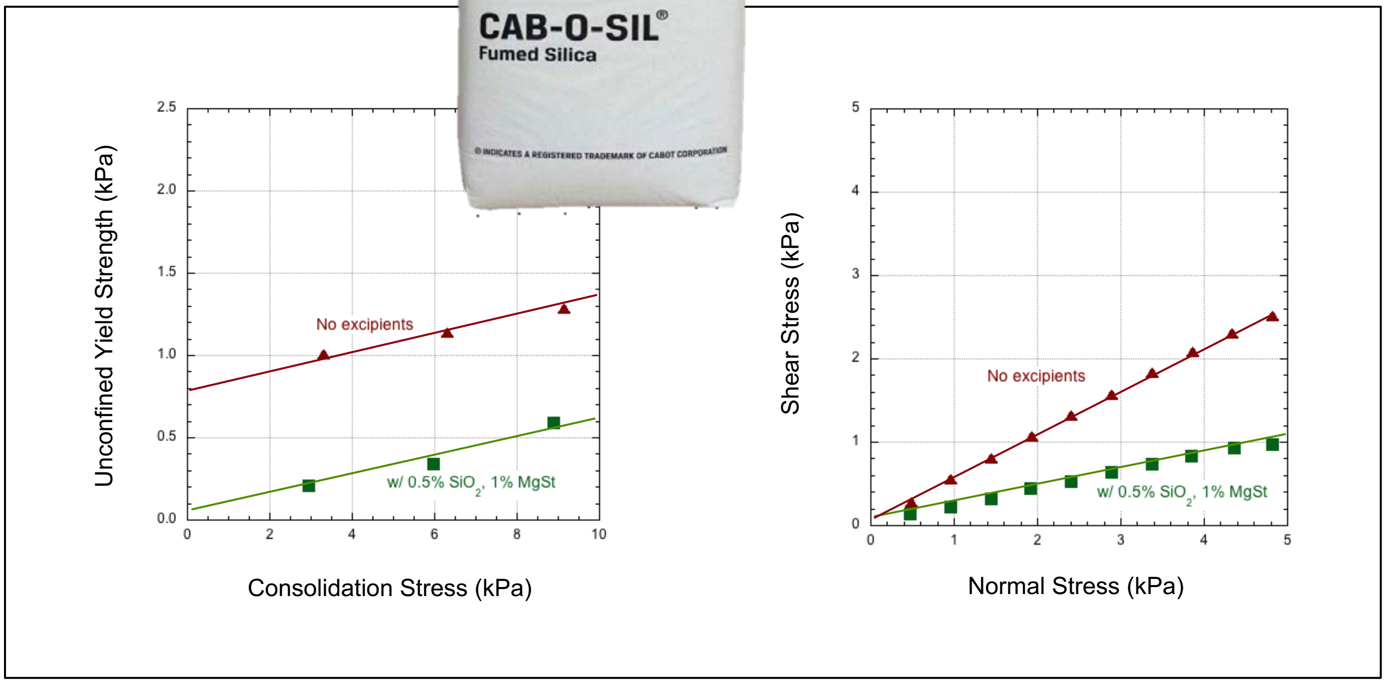


Example

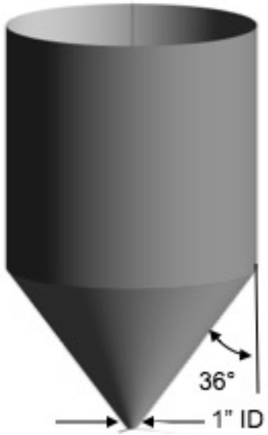


Example

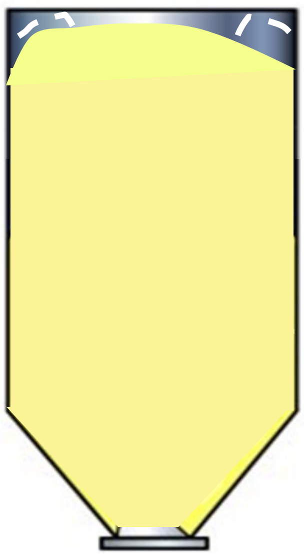

CABOT
CAB-O-SIL®
Fumed Silica


Unconfined Yield Strength (kPa)
Consolidation Stress (kPa)
No excipients
w/ 0.5% SiO2, 1% MgSt
Shear Stress (kPa)
Normal Stress (kPa)
No excipients
w/ 0.5% SiO2, 1% MgSt

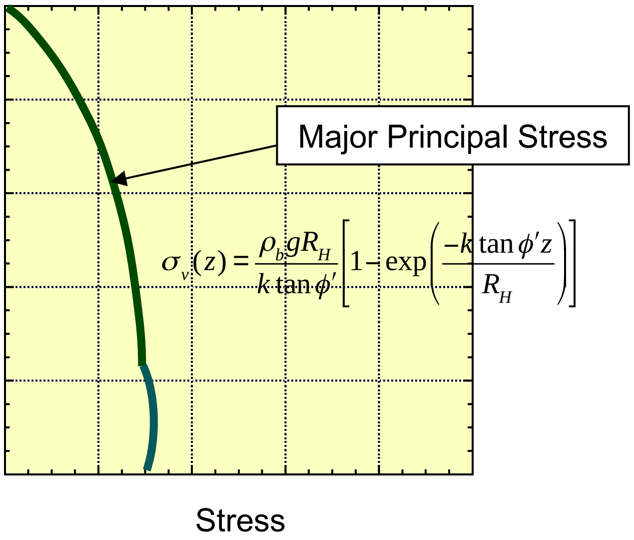
Example



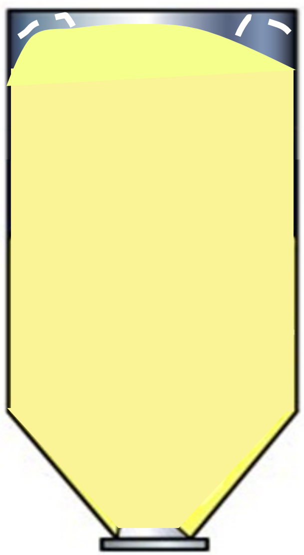
Funnel flow bins



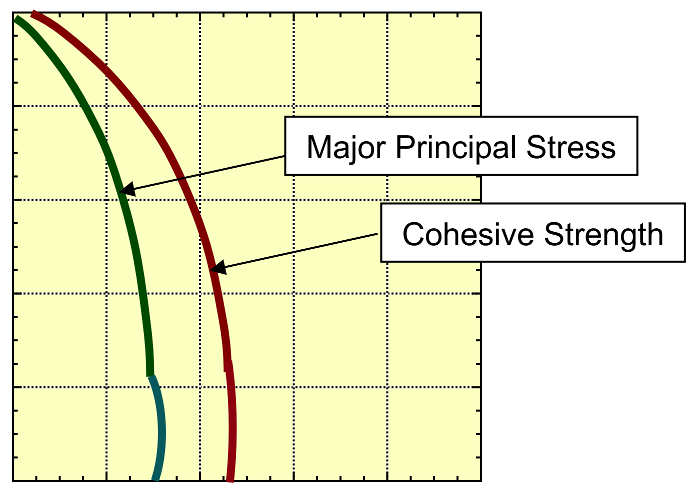
D = diameter of round outlet or
diagonal of slotted outlet



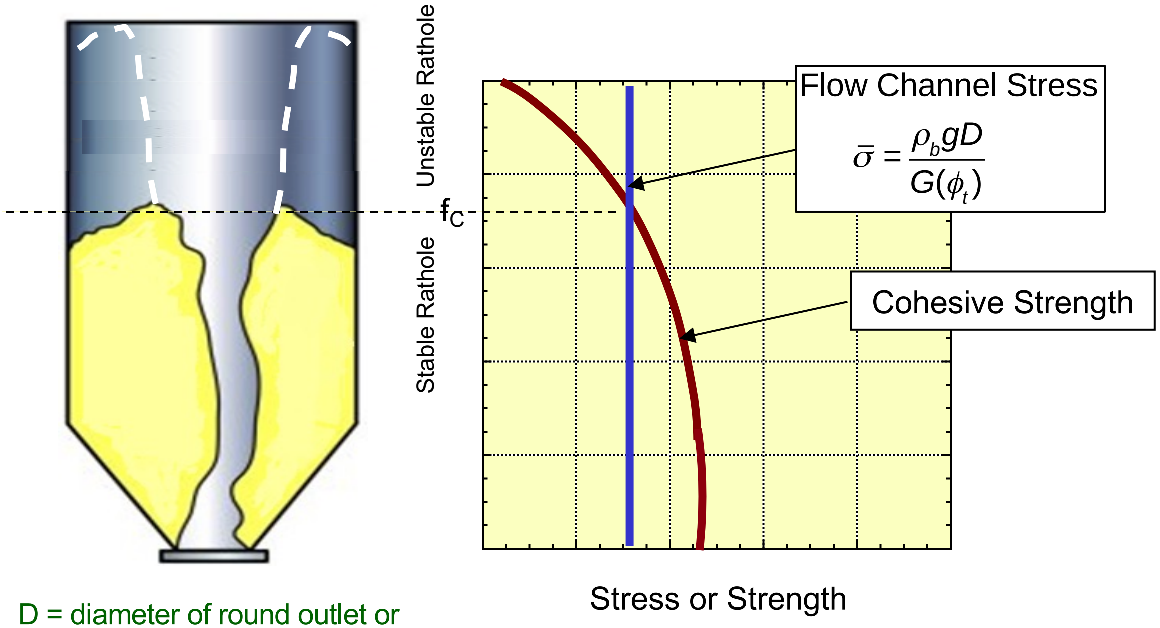
Funnel flow bins



D = diameter of round outlet or *diagonal* of slotted outlet



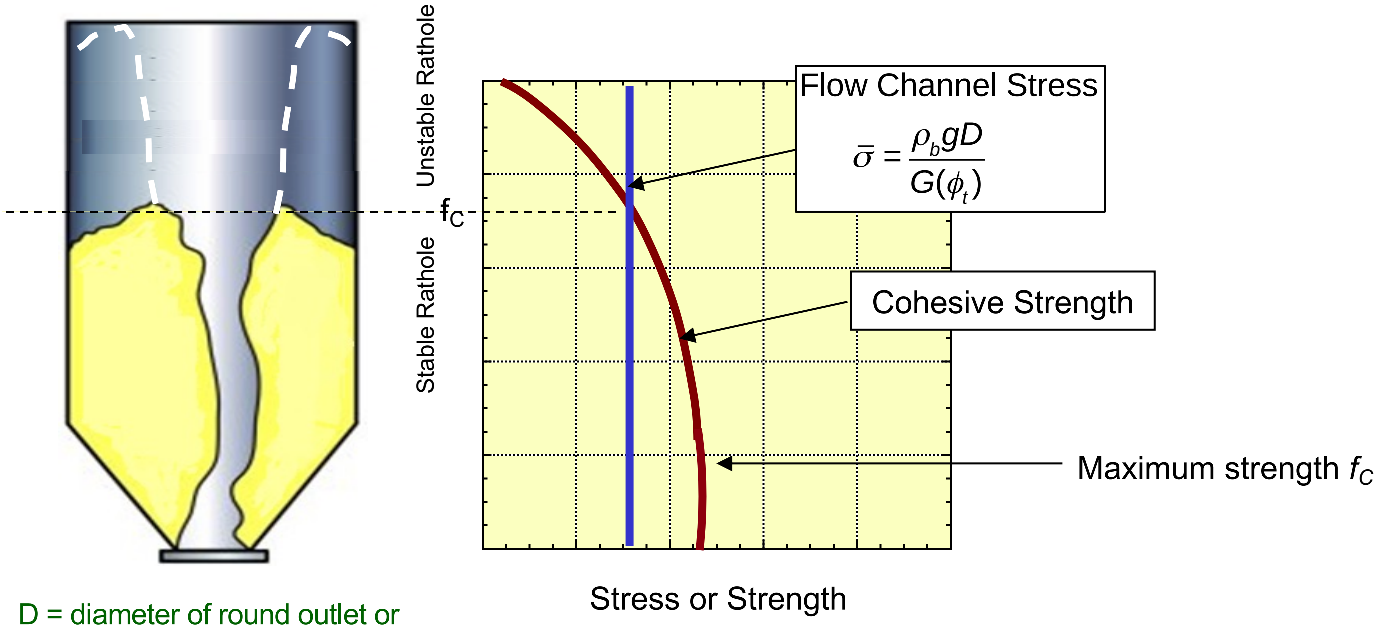
Funnel flow bins



D = diameter of round outlet or *diagonal* of slotted outlet

GM&A

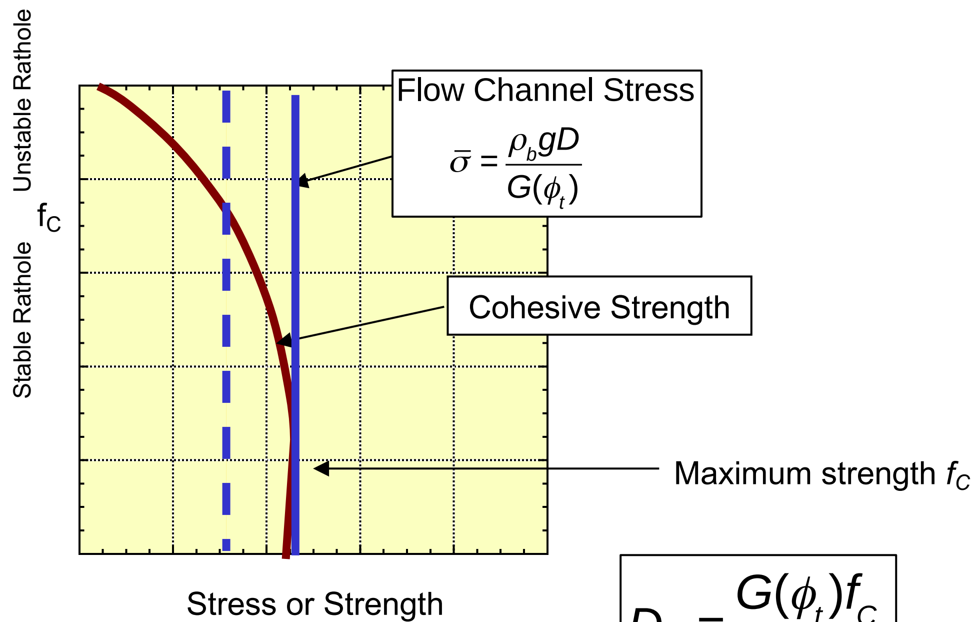
Funnel flow bins



D = diameter of round outlet or *diagonal* of slotted outlet

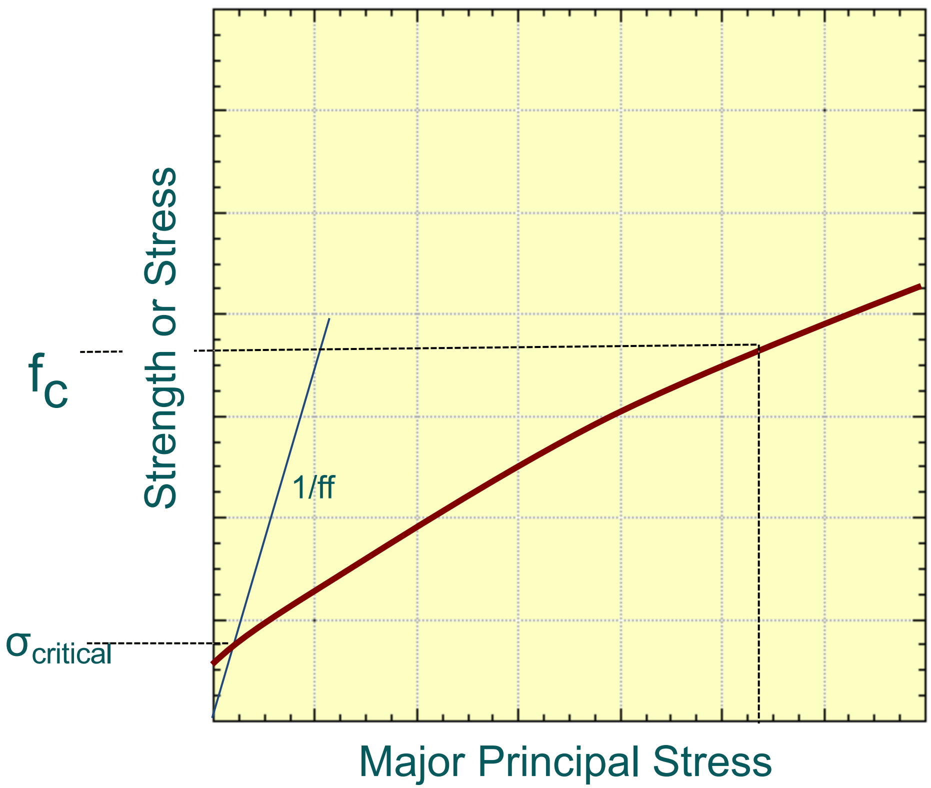
Funnel flow bins

D = diameter of round outlet or *diagonal* of slotted outlet



$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

Mass flow *vs.* funnel flow



To prevent arching in a mass flow bin:

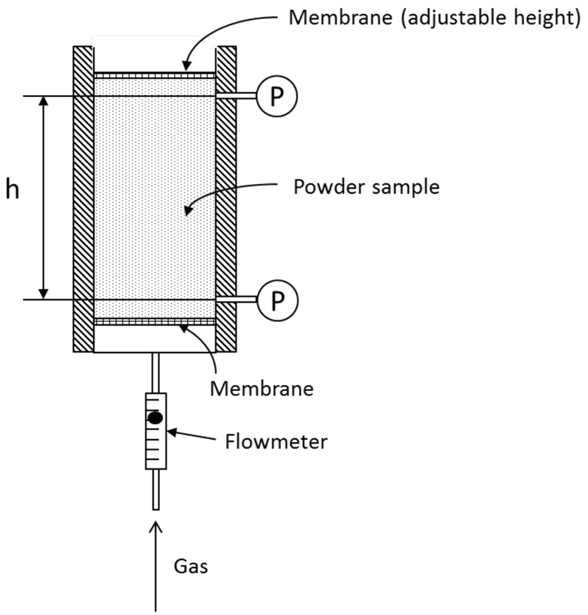
$$B_{min} = \frac{H(\theta')\sigma_{critical}}{\rho_b g}$$

To prevent a stable rathole in a funnel flow bin:

$$D_F = \frac{G(\phi_t) f_C}{\rho_b g}$$

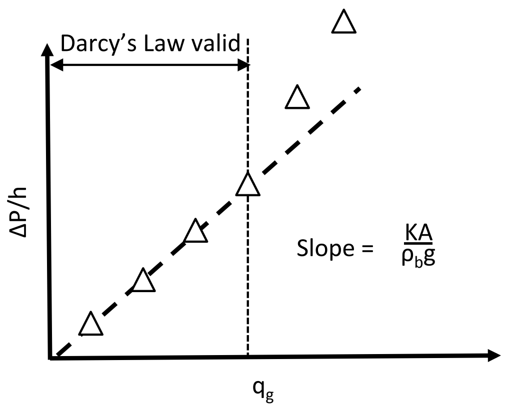
$$\sigma_v = \frac{\rho_b g R_H}{k \tan\phi'}\left[1 - \exp\left(\frac{-k \tan\phi' z}{R_H}\right)\right]$$

Permeability

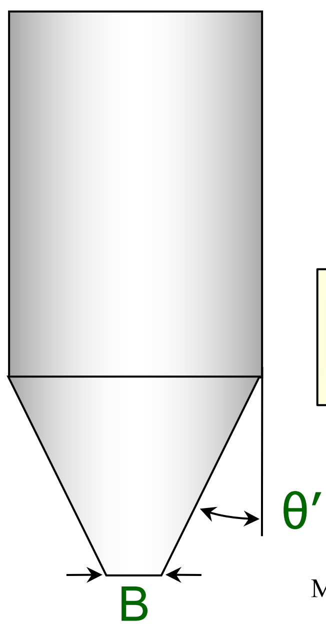


Darcy's Law

$$q_g = \frac{KA}{\rho_b g}\frac{\Delta P}{h}$$



Solids discharge rates



$$\frac{2(m+1)\tan\theta'}{Bg} v_o^2 = 1 - \frac{2(m+1)\cos\theta'\sin(\beta+\theta') f_c}{\rho_b g B} + \frac{1}{\rho_{bo} g}\left.\frac{dP}{dz}\right|_o$$

$$\left[\frac{2(m+1)\tan\theta'}{Bg}\right] v_o^2 + \left[\frac{1}{K_o}\left(1 - \frac{\rho_{bo}}{\rho_{bmp}}\right)\right] v_o + \frac{2(m+1)\cos\theta'\sin(\beta+\theta') f_c}{\rho_b g B} - 1 = 0$$

Permeability

Mehos, G., “Maximum Solids Discharge Rates from Hoppers”, *Chem. Engr. Res. Des.*, 191, 564 (2023)

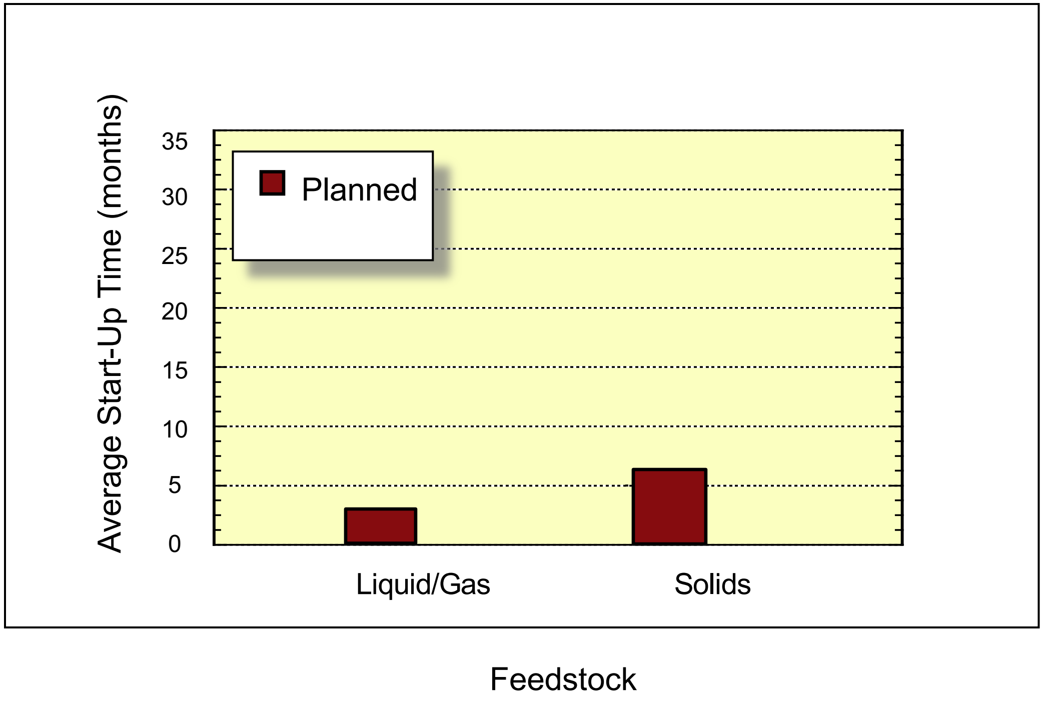
Summary

- Flow problems
  - Arching and ratholing
  - Caking
  - Segregation
  - Flooding and erratic discharge
- Flow properties
  - Cohesive strength
  - Internal friction
  - Compressibility
  - Wall friction
  - Permeability
- Design
  - Outlet dimension
  - Hopper angle

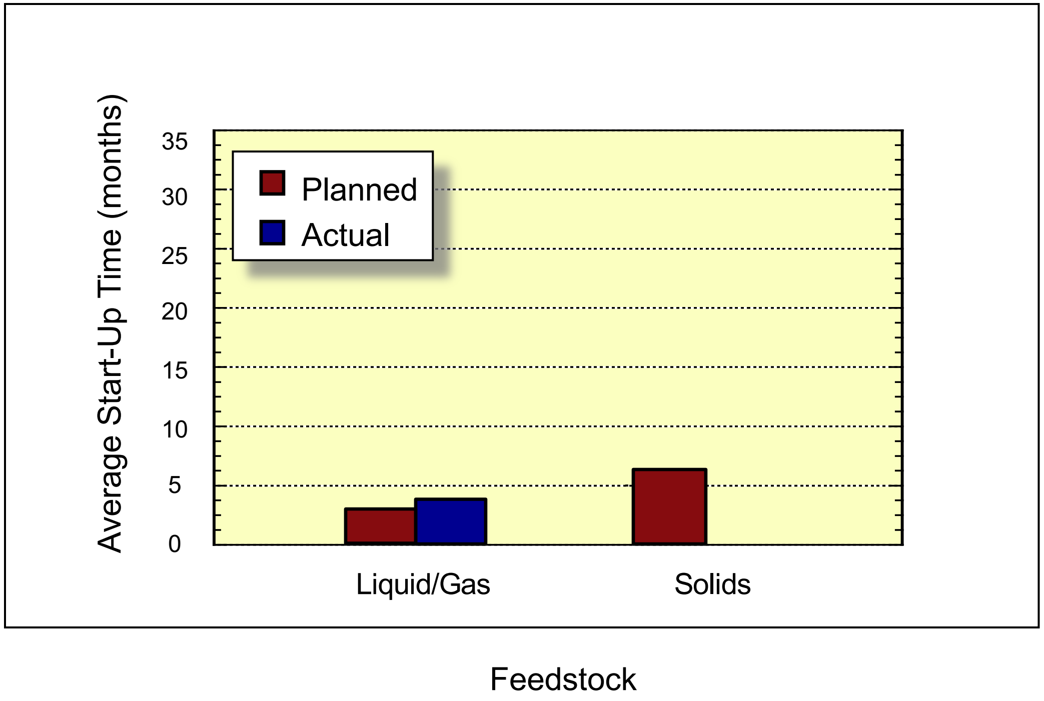
Rand Corporation Study

- Study by independent organization
- 40 solids processing plants in U.S. and Canada
- Six-year duration

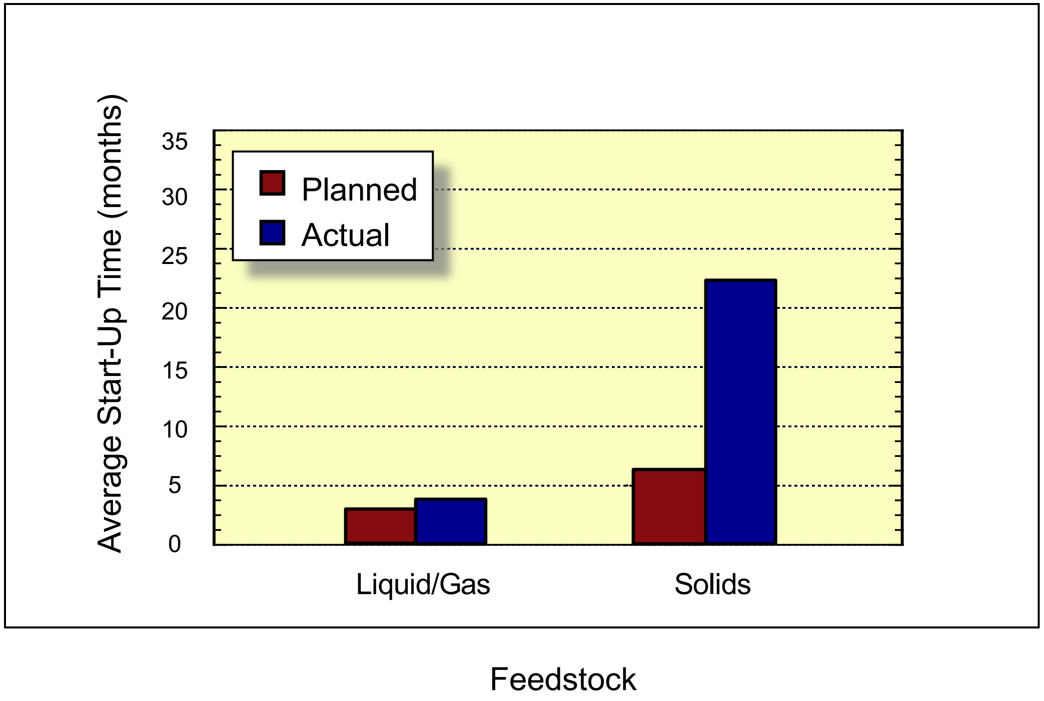
Planned vs. actual start-up



Planned vs. actual start-up

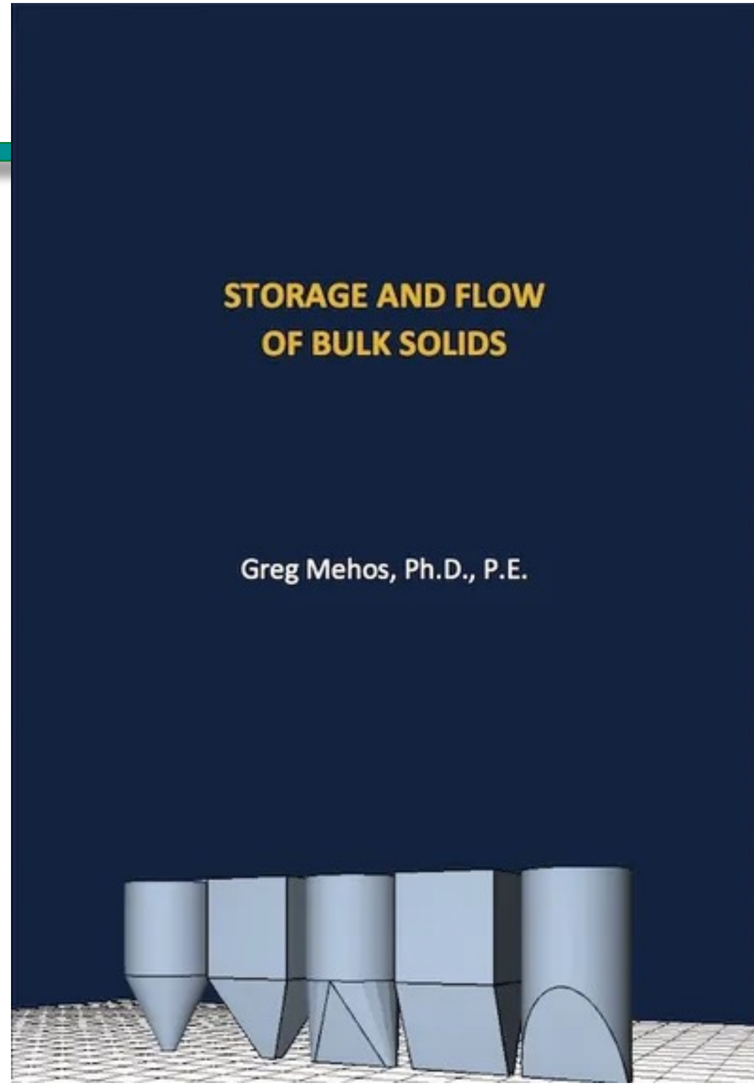


Planned vs. actual start-up



Rand conclusions

- 80% experienced solids handling problems
- Average startup time *ca.* 20 months (*vs.* 4 mo. for fluids)
- Typical performance 40 to 50% of design


STORAGE AND FLOW
OF BULK SOLIDS
Greg Mehos, Ph.D., P.E.

8. BETTER REFERENCES THAN THIS ONE

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Flow and Storage of Powders

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